DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :5

Topic :-TRIGONOMETRIC FUNCTIONS

781 **(a)**

We have,

$2\sin\theta = r^4 - 2r^2 + 3$

$\Rightarrow 2\sin\theta = (r^2-1)^2 + 2$

Clearly, LHS ≤ 2 and RHS ≥ 2

So, the equation is meaningful if each side is equal to 2

Clearly, RHS $= 2$ for $r^2 = 1$

For $r^2 = 1$, we have

$2\sin\theta = 2$

$\Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ $[\because 0 \leq \theta \leq 5\pi]$

Also, $r^2 = 1 \Rightarrow r = \pm 1$

Hence, the total number of pairs of the form (r, θ) is $2 \times 3 = 6$

783 **(a)**

We have,

$$\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2+b^2+c^2}{4\Delta^2}$$

Also,

$$\cot A + \cos B + \cot C = \frac{2R}{abc}(b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2)$$

$$\Rightarrow \cot A + \cos B + \cot C = \frac{R(a^2+b^2+c^2)}{abc} = \frac{a^2+b^2+c^2}{4\Delta}$$

Hence, $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$

784 **(c)**

We have,

$\sin 2\theta + 2 = 4\sin\theta + \cos\theta$

$\Rightarrow 2\sin\theta\cos\theta - \cos\theta + 2 - 4\sin\theta = 0$

$\Rightarrow \cos\theta(2\sin\theta - 1) - 2(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(\cos\theta - 2) = 0$

$\Rightarrow 2\sin\theta - 1 = 0$ $[\because \cos\theta - 2 \neq 0]$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}, 4\pi + \frac{\pi}{6}, 4\pi + \frac{5\pi}{6}$

Hence, the equation has 4 solutions

ALITER The curves $y = \sin x$ and $y = \frac{1}{2}$ intersect at 4 points in $[\pi, 5\pi]$. So, the equation has 4 solutions

785 **(c)**

For a triangle inscribed in a circle, we have

$$\frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = R$$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C$

$$= \frac{a^2}{4R^2} + \frac{b^2}{4R^2} + \frac{c^2}{4R^2}(a^2 + b^2 + c^2)$$

It is given that

$$\frac{a^2 + b^2 + c^2}{2} = 2(2R)^2 \Rightarrow a^2 + b^2 + c^2 = 16R^2$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \frac{1}{4R^2}(16R^2) = 4$$

786 **(d)**

We have,

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$$

$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$$

$$= 2\frac{\sin 54° - \sin 18°}{\sin 54° \sin 18°} = 2\frac{\cos 36° - \sin 18°}{\sin 18° \cos 36°} = 4$$

787 **(c)**

Given, $\sin 4A + \sin 2A = \cos 4A + \cos 2A$

$\Rightarrow 2\sin 3A \cos A = 2\cos 3A \cos A$

$\therefore \tan 3A = 1$ and $\cos A = 0$

$\Rightarrow A = \frac{\pi}{12}$ and $A = \frac{\pi}{2} \notin \left(0, \frac{\pi}{4}\right)$

$\therefore \tan 4A = \tan\frac{\pi}{3} = \sqrt{3}$

788 **(c)**

We have,

$$\sin A + \sin B = \frac{a+b}{c}$$

$$\Rightarrow \sin A + \sin B = \frac{\sin A + \sin B}{\sin C} \Rightarrow \sin C = 1$$

789 **(a)**

We have,

$\sin(\alpha+\beta) = 1, \sin(\alpha-\beta) = \frac{1}{2}$

$\Rightarrow \alpha+\beta = \frac{\pi}{2}$ and, $\alpha-\beta = \frac{\pi}{6}$

$\Rightarrow \alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$

$\therefore \tan(\alpha+2\beta)\tan(2\alpha+\beta)$

$= \tan\left(\frac{2\pi}{3}\right)\tan\frac{5\pi}{6} = \left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right) = 1$

790 **(c)**

Let $a_0 = \cos\theta$. Then,

$a_1 = \sqrt{\frac{1}{2}(1+a_0)} = \sqrt{\frac{1}{2}(1+\cos\theta)} = \cos\frac{\theta}{2}$

$a_2 = \sqrt{\frac{1}{2}(1+a_1)} = \sqrt{\frac{1}{2}\left(1+\cos\frac{\theta}{2}\right)} = \cos\left(\frac{\theta}{2^2}\right)$

and so on

Now, $\frac{1-a_0^2}{a_1 a_2 a_3 \ldots \text{to } \infty}$

$= \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}\ldots\text{to } \infty}$

$= \lim_{n\to\infty} \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}\ldots\text{to } \infty}$

$= \lim_{n\to\infty} \frac{\{2^n \sin(\theta/2^n)\}\sin\theta}{\sin(2^n \times \theta/2^n)} = \lim_{n\to\infty} \frac{\sin(\theta/2^n).\theta}{(\theta/2^n)} = \theta = a_0$

791 **(b)**

Le the angles of triangle ABC be $A=\theta, B = 2\,\theta$ and $C = 7\,\theta$. Then,

$A+B+C = 180° \Rightarrow 10\,\theta = 180° \Rightarrow \theta = 18°$

$\therefore A = 18°, B = 36°$ and $C = 126°$

Clearly, c is the greatest side and a is the smallest side.

Now,

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{c}{a} = \frac{\sin C}{\sin A} = \frac{\sin 126°}{\sin 18°} = \frac{\cos 36°}{\sin 18°} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

792 **(b)**

We have,

$A = \frac{2\,\pi}{3}$ and $\Delta = \frac{9\sqrt{3}}{2}\text{cm}^2$

$\therefore \Delta = \frac{1}{2}bc\sin A \Rightarrow \frac{9\sqrt{3}}{2} = \frac{1}{2}\,bc\sin\frac{2\,\pi}{3} \Rightarrow bc = 18$

Also,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow \cos\frac{2\pi}{3}=\frac{(b-c)^2+2bc-a^2}{2bc}$$

$$\Rightarrow -\frac{1}{2}=\frac{27+36-a^2}{36}\Rightarrow a^2=81\Rightarrow a=9\text{ cm}$$

793 **(b)**

We have,

$s=8k$ and, $\Delta=\sqrt{8k\times 3k\times 2k\times 3k}=12\,k^2$

$$\therefore r=\frac{\Delta}{s}\Rightarrow\frac{12\,k^2}{8k}=6\Rightarrow k=4$$

794 **(d)**

We have,

$$\frac{2^{\sin x}+2^{\cos x}}{2}\geq\sqrt{2^{\sin x}2^{\cos x}}\quad[\because AM\geq GM]$$

$$\Rightarrow 2^{\sin x}+2^{\cos x}\geq\sqrt{2^{\sin x+\cos x}}$$

$$\Rightarrow 2^{\sin x}+2^{\cos x}\geq 2\sqrt{2^{-\sqrt{2}}}\quad\left[\because -\sqrt{2}\leq\sin x+\cos x\leq\sqrt{2}\right]$$

$$\Rightarrow 2^{\sin x}+2^{\cos x}\geq 2^{1-\frac{1}{\sqrt{2}}}$$

795 **(a)**

Let $A=\frac{1}{3\sin\theta-4\cos\theta+7}$

Now, A will be minimum when $3\sin\theta-4\cos\theta+7$ is maximum

$\therefore$ Maximum value of

$3\sin\theta-4\cos\theta+7=\sqrt{3^2+4^2}+7=12$

$\therefore$ Minimum value of $\frac{1}{3\sin\theta-4\cos\theta+7}$ is $\frac{1}{12}$

796 **(b)**

We have,

$$\operatorname{cosec}\theta=\frac{p+q}{p-q}$$

Now,

$$\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)=\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}=\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$\Rightarrow\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)=\sqrt{\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)^2}=\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$\Rightarrow \cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \sqrt{\frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}} = \sqrt{\frac{\frac{p+q}{p-q}-1}{\frac{p+q}{p-q}+1}} = \sqrt{\frac{q}{p}}$$

797 **(a)**

$\sin t + \cos t = \frac{1}{5}$

$\Rightarrow \frac{2\tan\frac{t}{2}}{1+\tan^2\frac{t}{2}} + \frac{1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}} = \frac{1}{5}$

$\Rightarrow 10\tan\frac{t}{2} + 5 - 5\tan^2\frac{t}{2} = 1 + \tan^2\frac{t}{2}$

$\Rightarrow 6\tan^2\frac{t}{2} - 10\tan\frac{t}{2} - 4 = 0$

$\Rightarrow \left(6\tan\frac{t}{2}+2\right)\left(\tan\frac{t}{2}-2\right) = 0$

$\Rightarrow \tan\frac{t}{2} = \frac{-1}{3}, 2$ for , $0 < t < \pi$

$\tan\frac{t}{2} = 2$

798 **(a)**

We have,

$\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$

$\Rightarrow \sin\alpha\cos\alpha\left(\cos^2\alpha - \sin^2\alpha\right) > 0$

$\Rightarrow \cos\alpha\left(1-\tan^2\alpha\right) > 0$ $[\because \sin\alpha > 0 \text{ for } 0 < \alpha < \pi]$

$\Rightarrow \cos\alpha > 0$ and $1 - \tan^2\alpha > 0$

$\Rightarrow \cos\alpha < 0$ and $1 - \tan^2\alpha < 0$

$\Rightarrow \alpha \in (0, \pi/4)$ or, $\alpha \in (3\pi/4, \pi)$

799 **(a)**

We have, $\alpha + \beta + \gamma = \pi$

Now, $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \sin^2\alpha + \sin^2(\beta-\gamma)\sin(\beta+\gamma)$

$= \sin^2\alpha + \sin(\pi-\alpha)\sin(\beta+\gamma)$ $(\because \alpha+\beta-\gamma=\pi)$

$= \sin^2\alpha + \sin\alpha\ \sin(\beta+\gamma)$

$= \sin\alpha[\sin\alpha + \sin(\beta+\gamma)]$

$= \sin\alpha[\sin(\pi-(\beta-\gamma)) + \sin(\beta+\gamma)]$

$= \sin\alpha[\sin(\beta-\gamma) + \sin(\beta+\gamma)]$

$= \sin\alpha\,[2\sin\beta\cos\gamma]$

$= 2\sin\alpha\sin\beta\cos\gamma$

800 **(c)**

$\sec x \cos 5x = -1$

$\Rightarrow \cos 5x = -\cos x$

$\Rightarrow 5x = 2n\pi \pm (\pi - x)$

$\Rightarrow x = \dfrac{(2n+1)\pi}{6}$ or $\dfrac{(2n-1)\pi}{6}$

The possible values of x which lies in the interval $(0, 2\pi)$ are $\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6}$ and $\frac{11\pi}{6}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	A	C	C	D	C	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	D	A	B	A	A	A	C