DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

**Solutions**

Subject : MATHS
DPP No. :4

# Topic :-TRIGONOMETRIC FUNCTIONS

761 **(c)**

We have,

$a = \sin^4\theta + \cos^4\theta \leq \sin^2\theta + \cos^2\theta \leq 1$

Also,

$a = \sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta + \cos^2\theta$

$\Rightarrow a = \sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$

$\Rightarrow \sin^2 2\theta = 2(1-a) \Rightarrow 2(1-a) \leq 1 \Rightarrow a \geq \frac{1}{2}$

Hence, $\frac{1}{2} \leq a \leq 1$

762 **(a)**

Let $\sqrt{3}+1 = r\cos\alpha$ and $\sqrt{3}-1 = r\sin\alpha$, then

$r = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}$

$= \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}} = 2\sqrt{2}$

and $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$\Rightarrow \alpha = \frac{\pi}{12}$

The given equation reduces to

$2\sqrt{2}\cos(\theta - \alpha) = 2$

$\Rightarrow \cos\left(\theta - \frac{\pi}{12}\right) = \cos\frac{\pi}{4}$

$\Rightarrow \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$

763 **(d)**

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$= \frac{1}{\sqrt{10}} \cdot \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}}\sqrt{1 - \frac{1}{10}}$

$\left[\because \sin A = \frac{1}{\sqrt{10}}, \sin B = \frac{1}{\sqrt{5}}\right]$

$= \frac{1}{\sqrt{10}}\sqrt{\frac{4}{5}} + \frac{1}{\sqrt{5}}\sqrt{\frac{9}{10}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow A + B = \frac{\pi}{4}$

764 **(b)**

The given equation can be rewritten as

$\tan\theta(\sin\theta + \sqrt{3}) = 0$

$\Rightarrow \tan\theta = 0$, but $\sin\theta + \sqrt{3} \neq 0$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$

765 **(a)**

We have, $y = \sin\theta - \cos\theta$ and $\sin\theta - \cos\theta$ lies between $-\sqrt{2}$ and $+\sqrt{2}$

$\therefore -\sqrt{2} \leq y \leq \sqrt{2}$

766 **(c)**

Now, $\sin(\alpha - \beta) = \sin(\theta - \beta - (\theta - \alpha))$

$= \sin(\theta - \beta) = \cos(\theta - \alpha) - \cos(\theta - \beta)\sin(\theta - \alpha)$

$= ba - \sqrt{1 - b^2}\sqrt{1 - a^2}$

and $\cos(\alpha - \beta) = \cos(\theta - \beta - (\theta - \alpha))$

$= \cos(\theta - \beta)\cos(\theta - \alpha) + \sin(\theta - \beta)\sin(\theta - \alpha)$

$= a\sqrt{1 - b^2} + b\sqrt{1 - a^2}$

$\therefore \cos^2(\alpha - \beta) + 2ab\sin(\alpha - \beta)$

$= (a\sqrt{1 - b^2} + b\sqrt{1 - a^2})^2 + 2ab(ab - \sqrt{1 - a^2}\sqrt{1 - b^2})$

$= a^2 + b^2$

767 **(d)**

We have,

$8\sec^2\theta - 6\sec\theta + 1 = 0$

$\Rightarrow (4\sec\theta - 1)(2\sec\theta - 1) = 0$

$\Rightarrow \sec\theta = \frac{1}{4}, \sec\theta = \frac{1}{2}$

But, this is not possible as $|\sec\theta| \geq 1$

768 **(c)**

We have,

$x^3 - 13x^2 + 54x - 72 = 0$

$\Rightarrow (x - 3)(x^2 - 10x + 24) = 0$

$\Rightarrow (x-3)(x-4)(x-6)=0 \Rightarrow x=3,4,6$

Let $a=3, b=4$ and $c=6$

$\therefore \frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}=\frac{61}{144}$

769 **(b)**

$\cos^4\theta-\sin^4\theta=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$

$=\cos 2\theta=2\cos^2\theta-1$

770 **(b)**

$\cos 15° \cos 7\frac{1}{2}° \sin 7\frac{1}{2}°$

$=\frac{1}{2}\cos 15° \sin 15°=\frac{1}{4}\sin 30°$

$=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$

771 **(b)**

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}$

$=\frac{2}{\sqrt{\cos^2\theta}}=\frac{2}{|\cos\theta|}$

$=-\frac{2}{\cos\theta}=-2\sec\theta\left(\because \frac{\pi}{2}<\theta<\pi\right)$

772 **(c)**

$\cos^2 A(3-4\cos^2 A)^2+\sin^2 A(3-4\sin^2 A)^2$

$=(3\cos A-4\cos^3 A)^2+(3\sin A-4\sin^3 A)^2$

$=(-\cos 3A)^2+(\sin 3A)^2=1$

773 **(a)**

It is given that $A, B, C$ are in A.P.

$\therefore 2B=A+C$

$\Rightarrow 3B=A+B+C \Rightarrow 3B=180° \Rightarrow B=60°$

Also,

$b:c=\sqrt{3}:\sqrt{2}$

$\Rightarrow \frac{\sin B}{\sin C}=\frac{\sqrt{3}}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{3}}{2\sin C}=\frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \sin C=\frac{1}{\sqrt{2}} \Rightarrow C=45°$

$\therefore A=180°-(60°+45°)=75°$

774 **(b)**

We have, $\tan x=\frac{b}{a}$

$$\therefore \sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$$
$$=\sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}}+\sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$$
$$=\sqrt{\frac{1+\tan x}{1-\tan x}}+\sqrt{\frac{1-\tan x}{1+\tan x}}$$
$$=\sqrt{\frac{\cos x+\sin x}{\cos x-\sin x}}+\sqrt{\frac{\cos x-\sin x}{\cos x+\sin x}}$$
$$=\frac{\cos x+\sin x+\cos x-\sin x}{\sqrt{\cos^2 x-\sin^2 x}}=\frac{2\cos x}{\sqrt{\cos 2x}}$$

775 **(c)**

We have,

$\sin A+\cos A=m$ and $\sin^3 A+\cos^3 A=n$

Now, $\sin A+\cos A=m$

$\Rightarrow(\sin A+\cos A)^3=m^3$

$\Rightarrow \sin^3 A+\cos^3 A+3\sin A\cos A(\sin A+\cos A)=m^3$

$\Rightarrow n+3\sin A\cos A\, m=m^3$ ...(i)

Again,

$\sin A+\cos A=m$

$\Rightarrow \sin^2 A+\cos^2 A+2\sin A\cos A=m^2$

$\Rightarrow \sin A\cos A=\frac{m^2-1}{2}$ ...(ii)

From (i) and (ii), we have

$$n+3m\frac{(m^2-1)}{2}=m^3$$

$\Rightarrow 2n+3m^3-3m=2m^3 \Rightarrow m^3-3m+2n=0$

776 **(b)**

$\because \sec x-1=(\sqrt{2}-1)\tan x$

$\Rightarrow 1-\cos x=(\sqrt{2}-1)\sin x$

$$\Rightarrow \sin\frac{x}{2}\left\{\sin\frac{x}{2}-(\sqrt{2}-1)\cos\frac{x}{2}\right\}=0$$

$$\Rightarrow \sin\frac{x}{2}=0 \text{ or } \tan\frac{x}{2}=\sqrt{2}-1=\tan\frac{\pi}{8}$$

$$\Rightarrow \frac{x}{2}=n\pi \text{ or } \frac{x}{2}=n\pi+\frac{\pi}{8}$$

$$\therefore x=2n\pi, 2n\pi+\frac{\pi}{4}$$

777 **(a)**

$\alpha-\beta=(\theta-\beta)-(\theta-\alpha)$

$\therefore \cos(\alpha-\beta)=\cos(\theta-\beta)\cos(\theta-\alpha)+\sin(\theta-\beta)\sin(\theta-\alpha)$

And $\sin(\alpha-\beta) = \sin(\theta-\beta)\cos(\theta-\alpha) - \sin(\theta-\alpha)\cos(\theta-\beta)$

$\Rightarrow \cos(\alpha-\beta) = b.a + \sqrt{1-a^2}\sqrt{1-b^2}$

And $\sin(\alpha-\beta) = \left(a\sqrt{1-b^2}\right) - \left(b\sqrt{1-b^2}\right)$

Now, $\sin^2(\alpha-\beta) = \left(a\sqrt{1-b^2}\right)^2 + \left(b\sqrt{1-b^2}\right)^2 - 2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$\Rightarrow \sin^2(\alpha-\beta) = a^2(1-b^2) + b^2(1-a^2) - 2ab\{\cos(\alpha-\beta) - ab\}$

$\Rightarrow \sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) = a^2 - a^2b^2 + b^2 - b^2a^2 + 2a^2b^2$

$\Rightarrow \sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) = a^2 + b^2$

778 **(a)**

We have, $S = \sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta$

We know that,

$\sin\theta + \sin(\theta+\beta) + \sin(\theta+2\beta) + \ldots n$ terms

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left[\frac{\theta+\theta+(n-1)\beta}{2}\right]$$

Put, $\beta = \theta$

$$\therefore S = \frac{\sin\frac{n\theta}{2}\cdot\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}$$

780 **(c)**

Given, $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$

$\Rightarrow \tan 3\theta - 1 - \sqrt{3}\tan 3\theta - \sqrt{3} = 0$

$\Rightarrow \tan 3\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \tan(45° + 60°)$

$\Rightarrow \tan 3\theta = \tan\frac{7\pi}{12}$

$\Rightarrow 3\theta = n\pi + \frac{7\pi}{12}$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{7\pi}{12}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | C | A | D | B | A | C | D | C | B | B |

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|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | C | A | B | C | B | A | A | C | C |
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