

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :3

Topic :-TRIGONOMETRIC FUNCTIONS

741 (a)

It is given that A, B, C are in A.P.

$$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$$

$$\Rightarrow \cos B = \frac{1}{2}$$

$$\Rightarrow \frac{c^2 + a^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow c^2 + a^2 - b^2 = ac$$

$$\Rightarrow (a - c)^2 = b^2 - ac$$

$$\Rightarrow |a - c| = \sqrt{b^2 - ac}$$

$$\Rightarrow |\sin A - \sin C| = \sqrt{\sin^2 B - \sin A \sin C}$$

$$\Rightarrow 2 \left| \sin \frac{A - C}{2} \right| \cos \frac{A + C}{2} = \sqrt{\frac{3}{4} - \sin A \sin C}$$

$$\Rightarrow 2 \left| \sin \frac{A - C}{2} \right| = \sqrt{3 - 4 \sin A \sin C}$$

$$\Rightarrow \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \frac{2 \left| \sin \frac{A - C}{2} \right|}{|A - C|}$$

$$\Rightarrow \lim_{A \rightarrow C} \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \lim_{A \rightarrow C} \left| \frac{\sin \left(\frac{A - C}{2} \right)}{\frac{A - C}{2}} \right| = 1$$

742 (c)

$$3 - \cos \theta + \cos \left(\theta + \frac{\pi}{3} \right)$$

$$= 3 - \cos \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$$

$$= 3 - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = 3 - \sin \left(\theta + \frac{\pi}{6} \right)$$

Since, $-1 \leq \sin \theta \leq 1$

Hence, the value of expression lies in $[2, 4]$

743

(c)

We have, $\cos A = m \cos B$

$$\Rightarrow \frac{\cos A}{\cos B} = \frac{m}{1}$$

$$\Rightarrow \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2 \cos \frac{A+B}{2} \cos \frac{B-A}{2}}{2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot \frac{A+B}{2} = \left(\frac{m+1}{m-1} \right) \tan \frac{B-A}{2}$$

But $\cot \frac{A+B}{2} = \lambda \tan \frac{B-A}{2}$

$$\therefore \lambda = \frac{m+1}{m-1}$$

744

(c)

$$\begin{aligned} & \cos^4 \frac{\pi}{8} + \cos^4 \frac{7\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} \\ &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \cos^4 \left(\frac{\pi}{2} + \frac{\pi}{8} \right) \\ &= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] \\ &= 2 \left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right] \\ &= 2 \left[1 - \frac{1}{2} \left(\sin \frac{\pi}{4} \right)^2 \right] \\ &= 2 \left[1 - \frac{1}{4} \right] = \frac{3}{2} \end{aligned}$$

745

(b)

Given, $\sin \theta = \frac{12}{13}$ and $\cos \phi = -\frac{3}{5}$

$$\therefore \cos \theta = \frac{5}{13} \text{ and } \sin \phi = -\frac{4}{5}$$

$$\therefore \sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$= \frac{12}{13} \times \left(-\frac{3}{5} \right) + \frac{5}{13} \times \left(-\frac{4}{5} \right)$$

$$= \frac{-36}{65} + \frac{(-20)}{65} = -\frac{56}{65}$$

746

(c)

We have,

$$\sec^2 \theta \operatorname{cosec}^2 \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} - \frac{4}{\sin^2 2\theta} \geq 4$$

$$\text{and, } \sec^2 \theta \operatorname{cosec}^2 \theta = \frac{4}{\sin^2 2\theta} \geq 4$$

Thus, the required equation is

$$x^2 - \lambda x + \lambda = 0, \text{ where } \lambda \geq 4$$

747

(a)

$$\frac{1}{m} \left[(m+n) + \frac{1}{(m+n)} \right]$$

$$= \frac{1}{\sec \theta} \left[\sec \theta + \tan \theta + \frac{1}{\sec \theta + \tan \theta} \right]$$

$$= \frac{[\sec^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta + 1]}{\sec \theta (\sec \theta + \tan \theta)}$$

$$= \frac{2 \sec^2 \theta + 2 \sec \theta \tan \theta}{\sec \theta (\sec \theta + \tan \theta)}$$

$$= 2$$

748

(a)

$$\because \sin A \sin B = \frac{1}{2} \times 2 \sin A \sin B$$

$$= \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$= \frac{1}{2} [\cos(A-B) - \cos 90^\circ] \quad (\because A+B+C=180^\circ \text{ and } \angle C=90^\circ, \text{ given})$$

$$= \frac{1}{2} \cos(A-B) \leq \frac{1}{2}$$

$$\therefore \text{Maximum value of } \sin A \sin B = \frac{1}{2}$$

749

(b)

In cyclic quadrilateral $ABCD$, we have

$$A+C=\pi \text{ and } B+D=\pi$$

$$\therefore \cos A = -\cos C \text{ and } \cos B = -\cos D$$

$$\Rightarrow \cos A + \cos B + \cos C + \cos D = 0$$

750

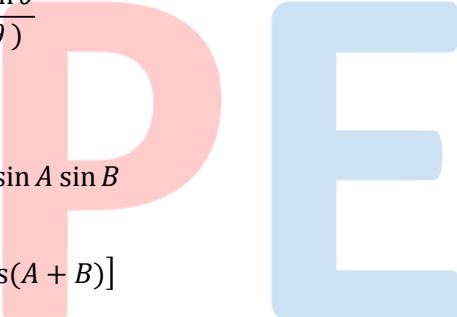
(d)

Let $A = \theta, B = 2\theta$ and $C = 3\theta$. Then,

$$A+B+C=180^\circ \Rightarrow 6\theta=180^\circ \Rightarrow \theta=30^\circ$$

$$\therefore A=30^\circ, B=60^\circ \text{ and } C=90^\circ$$

Now,



$$a:b:c = \sin A : \sin B : \sin C,$$

$$\Rightarrow a:b:c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 \Rightarrow a:b:c = 1:\sqrt{3}:2$$

751

(b)

We have,

$$\sin(\pi \cos \theta) = \cos(\pi \sin \theta)$$

$$\Rightarrow \sin(\pi \cos \theta) = \sin\left(\frac{\pi}{2} + \pi \sin \theta\right)$$

$$\Rightarrow \pi \cos \theta = \frac{\pi}{2} + \pi \sin \theta$$

$$\Rightarrow \pi \cos \theta - \pi \sin \theta = \frac{\pi}{2}$$

$$\Rightarrow \cos \theta - \sin \theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2\sqrt{2}} \Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

752

(a)

The given equation is not meaningful for

$$|\cos x| = 1$$

So, let $|\cos x| \neq 1$

Now,

$$2^{1+|\cos x|+\cos^2 x+|\cos x|^3+\dots+\text{to } \infty} = 4$$

$$\Rightarrow \frac{1}{2^{1-|\cos x|}} = 2^2$$

$$\Rightarrow \frac{1}{1-|\cos x|} = 2$$

$$\Rightarrow 2 - 2|\cos x| = 1$$

$$\Rightarrow |\cos x| = \frac{1}{2}$$

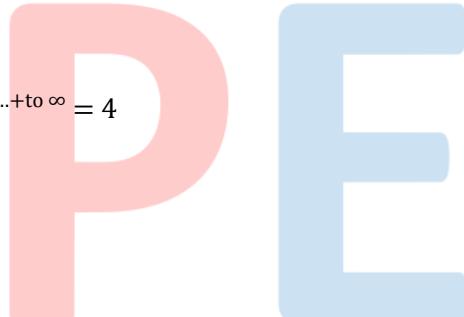
$$\Rightarrow \cos x = \pm \frac{1}{2}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{3}, \cos x = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = (2n \pm 1)\pi \pm \frac{\pi}{3}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$



753

(b)

We have,

$$(\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$$

$$\Rightarrow (\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta) - (\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta) = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$$

754 (d)

The given expression can be written as $\frac{1 + \cos y - \sin^2 y}{1 + \cos y} + \frac{(1 - \cos^2 y) - \sin^2 y}{\sin y(1 - \cos y)}$

$$= \frac{\cos y(1 + \cos y)}{1 + \cos y} + 0 = \cos y$$

755 (a)

We have,

$$a = 2b$$

$$\Rightarrow 2R \sin A = 4R \sin B$$

$$\Rightarrow \sin A = 2 \sin B$$

$$\Rightarrow \sin 3B = 2 \sin B \quad [\because A = 3B]$$

$$\Rightarrow 3 \sin B - 4 \sin^3 B = 2 \sin B$$

$$\Rightarrow \sin B - 4 \sin^3 B = 0$$

$$\Rightarrow 1 - 4 \sin^2 B = 0 \Rightarrow \sin B = \frac{1}{2} \Rightarrow B = \frac{\pi}{6}$$

$$\therefore A = 3B = \frac{\pi}{2}$$

756 (b)

We know that

$$AD^2 = \frac{1}{4}(b^2 + c^2 + 2bc \cos A)$$

$$\therefore 4AD^2 = b^2 + c^2 + 2bc \cos \frac{\pi}{3} \Rightarrow 4AD^2 = b^2 + c^2 + bc$$

757 (a)

We know that,

$$\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$$

$$\therefore \cos(\alpha - \beta) = \cos(\theta - \beta) \cos(\theta - \alpha) + \sin(\theta - \beta) \sin(\theta - \alpha)$$

$$= ab + \sqrt{1 - a^2} \sqrt{1 - b^2}$$

$$\text{and } \sin(\alpha - \beta) = \pm (a\sqrt{1 - b^2} - b\sqrt{1 - a^2})$$

$$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2 - 2ab\sqrt{1 - a^2}\sqrt{1 - b^2}$$

$$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2 - 2ab[\cos(\alpha - \beta) - ab]$$

$$\therefore \sin^2(\alpha - \beta) = a^2 + b^2 - 2ab \cos(\alpha - \beta)$$

$$\Rightarrow \sin^2(\alpha - \beta) + 2ab \cos(\alpha - \beta) = a^2 + b^2$$

758 (b)

$$\frac{1}{2} \tan \frac{x}{2} = \frac{1}{2} \cot \frac{x}{2} - \cot x \quad \left[\because \cot x = \frac{1 - \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2}} \right]$$

$$\text{And } \frac{1}{2^2} \tan \frac{x}{2^2} = \frac{1}{2^2} \cot \left(\frac{x}{2^2} \right) - \frac{1}{2} \cot \left(\frac{x}{2} \right)$$

$$\text{Similarly, } \frac{1}{2^3} \tan \left(\frac{x}{2^3} \right) = \frac{1}{2^3} \cot \left(\frac{x}{2^3} \right) - \frac{1}{2^2} \cot \dots \left(\frac{x}{2^2} \right) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\frac{1}{2^n} \tan \left(\frac{x}{2^n} \right) = \frac{1}{2^n} \cot \left(\frac{x}{2^n} \right) - \frac{1}{2^{n-1}} \cot \left(\frac{x}{2^{n-1}} \right)$$

On adding all the above results, we get

$$\frac{1}{2} \tan \frac{x}{2} + \frac{1}{2^2} \tan \left(\frac{x}{2^2} \right) + \dots + \frac{1}{2^n} \tan \left(\frac{x}{2^n} \right) = \frac{1}{2^n} \cot \left(\frac{x}{2^n} \right) - \cot x$$

759

(c)

It is given that

Area of ΔABC = Area of ΔDEF

$$\Rightarrow \frac{1}{2} AB \cdot AC \sin A = \frac{1}{2} CE \cdot EF \sin E$$

$$\Rightarrow \sin A = \sin E$$

$$\Rightarrow \sin 2E = \sin E$$

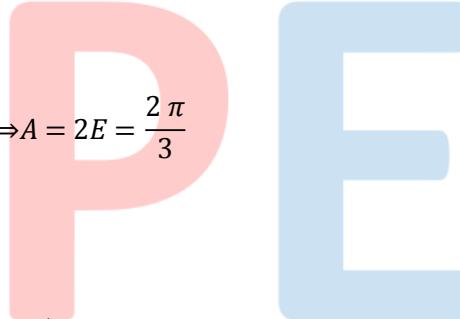
$$\Rightarrow 2E = \pi - E \Rightarrow E = \frac{\pi}{3} \Rightarrow A = 2E = \frac{2\pi}{3}$$

760

(c)

We have,

$$\begin{aligned} & \sin \frac{\pi}{18} \sin \frac{5\pi}{18} \sin \frac{7\pi}{18} \\ &= \cos \left(\frac{\pi}{2} - \frac{\pi}{18} \right) \cos \left(\frac{\pi}{2} - \frac{5\pi}{18} \right) \cos \left(\frac{\pi}{2} - \frac{7\pi}{18} \right) \\ &= \cos \frac{8\pi}{18} \cos \frac{4\pi}{18} \cos \frac{2\pi}{8} \\ &= \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} = \frac{\sin (2^3 \cdot \pi/9)}{2^3 \sin \pi/9} = \frac{1}{2^3} = \frac{1}{8} \end{aligned}$$



ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	C	C	B	C	A	A	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	D	A	B	A	B	C	C

PE