DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :3

Topic :-TRIGONOMETRIC FUNCTIONS

741 **(a)**

It is given that A,B,C are in A.P.

$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180° \Rightarrow B = 60°$

$\Rightarrow \cos B = \frac{1}{2}$

$\Rightarrow \frac{c^2 + a^2 - b^2}{2ac} = \frac{1}{2}$

$\Rightarrow c^2 + a^2 - b^2 = ac$

$\Rightarrow (a - c)^2 = b^2 - ac$

$\Rightarrow |a - c| = \sqrt{b^2 - ac}$

$\Rightarrow |\sin A - \sin C| = \sqrt{\sin^2 B - \sin A \sin C}$

$\Rightarrow 2\left|\sin \frac{A - C}{2}\right| \cos \frac{A + C}{2} = \sqrt{\frac{3}{4} - \sin A \sin C}$

$\Rightarrow 2\left|\sin \frac{A - C}{2}\right| = \sqrt{3 - 4 \sin A \sin C}$

$\Rightarrow \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \frac{2\left|\sin \frac{A - C}{2}\right|}{|A - C|}$

$\Rightarrow \lim_{A \to C} \frac{\sqrt{3 - 4 \sin A \sin C}}{|A - C|} = \lim_{A \to C} \left| \frac{\sin\left(\frac{A - C}{2}\right)}{\frac{A - C}{2}} \right| = 1$

742 **(c)**

$3 - \cos \theta + \cos\left(\theta + \frac{\pi}{3}\right)$

$= 3 - \cos \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$

$= 3 - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = 3 - \sin\left(\theta + \frac{\pi}{6}\right)$

Since, $-1 \leq \sin\theta \leq 1$

Hence, the value of expression lies in [2, 4]

743 **(c)**

We have, $\cos A = m\cos B$

$$\Rightarrow \frac{\cos A}{\cos B} = \frac{m}{1}$$

$$\Rightarrow \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2\cos\frac{A+B}{2}\cos\frac{B-A}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot\frac{A+B}{2} = \left(\frac{m+1}{m-1}\right)\tan\frac{B-A}{2}$$

But $\cot\frac{A+B}{2} = \lambda\tan\frac{B-A}{2}$

$$\therefore\ \lambda = \frac{m+1}{m-1}$$

744 **(c)**

$$\cos^4\frac{\pi}{8} + \cos^4\frac{7\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8}$$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right) + \cos^4\left(\frac{\pi}{2} + \frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right]$$

$$= 2\left[\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right]$$

$$= 2\left[1 - \frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right]$$

$$= 2\left[1 - \frac{1}{4}\right] = \frac{3}{2}$$

745 **(b)**

Given, $\sin\theta = \frac{12}{13}$ and $\cos\phi = -\frac{3}{5}$

$\therefore\ \cos\theta = \frac{5}{13}$ and $\sin\phi = -\frac{4}{5}$

$\therefore\ \sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$= \frac{12}{13} \times \left(-\frac{3}{5}\right) + \frac{5}{13} \times \left(-\frac{4}{5}\right)$$

$$= \frac{-36}{65} + \frac{(-20)}{65} = -\frac{56}{65}$$

746 **(c)**

We have,

$$\sec^2\theta \operatorname{cosec}^2\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} - \frac{4}{\sin^2 2\theta} \geq 4$$

and, $\sec^2\theta \operatorname{cosec}^2\theta = \frac{4}{\sin^2 2\theta} \geq 4$

Thus, the required equation is

$x^2 - \lambda x + \lambda = 0$, where $\lambda \geq 4$

747 **(a)**

$$\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right]$$

$$= \frac{1}{\sec\theta}\left[\sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}\right]$$

$$= \frac{[\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1]}{\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta(\sec\theta + \tan\theta)}$$

$= 2$

748 **(a)**

$$\because \sin A \sin B = \frac{1}{2} \times 2\sin A \sin B$$

$$= \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$= \frac{1}{2}[\cos(A - B) - \cos 90°]$ $(\because A + B + C = 180°$ and $\angle C = 90°$, given)

$$= \frac{1}{2}\cos(A - B) \leq \frac{1}{2}$$

$\therefore$ Maximum value of $\sin A \sin B = \frac{1}{2}$

749 **(b)**

In cyclic quadrilateral $ABCD$, we have

$A + C = \pi$ and $B + D = \pi$

$\therefore \cos A = -\cos C$ and $\cos B = -\cos D$

$\Rightarrow \cos A + \cos B + \cos C + \cos D = 0$

750 **(d)**

Let $A = \theta, B = 2\theta$ and $C = 3\theta$. Then,

$A + B + C = 180° \Rightarrow 6\theta = 180° \Rightarrow \theta = 30°$

$\therefore A = 30°, B = 60°$ and $C = 90°$

Now,

$a:b:c = \sin A : \sin B : \sin C,$

$\Rightarrow a:b:c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 \Rightarrow a:b:c = 1 : \sqrt{3} : 2$

751 **(b)**

We have,

$\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$

$\Rightarrow \sin(\pi \cos\theta) = \sin\left(\frac{\pi}{2} + \pi \sin\theta\right)$

$\Rightarrow \pi \cos\theta = \frac{\pi}{2} + \pi \sin\theta$

$\Rightarrow \pi \cos\theta - \pi \sin\theta = \frac{\pi}{2}$

$\Rightarrow \cos\theta - \sin\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}} \Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

752 **(a)**

The given equation is not meaningful for

$|\cos x| = 1$

So, let $|\cos x| \neq 1$

Now,

$2^{1+|\cos x| + \cos^2 x + |\cos x|^3 + \ldots + \text{to } \infty} = 4$

$\Rightarrow \frac{1}{2^{1-|\cos x|}} = 2^2$

$\Rightarrow \frac{1}{1 - |\cos x|} = 2$

$\Rightarrow 2 - 2|\cos x| = 1$

$\Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow \cos x = \pm\frac{1}{2}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}, \cos x = \cos\frac{2\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = (2n \pm 1)\pi \pm \frac{\pi}{3}$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$

753 **(b)**

We have,

$(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$

$\Rightarrow (\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta) - (\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta) = 0$

$\Rightarrow \cos 2\alpha + \cos 2\beta = -2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$\Rightarrow \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

754 **(d)**

The given expression can be written as $\frac{1+\cos y-\sin^2 y}{1+\cos y}+\frac{(1-\cos^2 y)-\sin^2 y}{\sin y(1-\cos y)}$

$$=\frac{\cos y\,(1+\cos y)}{1+\cos y}+0=\cos y$$

755 **(a)**

We have,

$a=2b$

$\Rightarrow 2R\sin A=4R\sin B$

$\Rightarrow \sin A=2\sin B$

$\Rightarrow \sin 3B=2\sin B \quad [\because A=3B]$

$\Rightarrow 3\sin B-4\sin^3 B=2\sin B$

$\Rightarrow \sin B-4\sin^3 B=0$

$\Rightarrow 1-4\sin^2 B=0 \Rightarrow \sin B=\frac{1}{2}\Rightarrow B=\frac{\pi}{6}$

$\therefore A=3B=\frac{\pi}{2}$

756 **(b)**

We know that

$AD^2=\frac{1}{4}(b^2+c^2+2bc\cos A)$

$\therefore 4AD^2=b^2+c^2+2bc\cos\frac{\pi}{3}\Rightarrow 4AD^2=b^2+c^2+bc$

757 **(a)**

We know that,

$\alpha-\beta=(\theta-\beta)-(\theta-\alpha)$

$\therefore \cos(\alpha-\beta)=\cos(\theta-\beta)\cos(\theta-\alpha)+\sin(\theta-\beta)\sin(\theta-\alpha)$

$=ab+\sqrt{1-a^2}\sqrt{1-b^2}$

and $\sin(\alpha-\beta)=\pm(a\sqrt{1-b^2}-b\sqrt{1-a^2})$

$\Rightarrow \sin^2(\alpha-\beta)=a^2+b^2-2a^2b^2-2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$\Rightarrow \sin^2(\alpha-\beta)=a^2+b^2-2a^2b^2-2ab[\cos(\alpha-\beta)-ab]$

$\therefore \sin^2(\alpha-\beta)-a^2+b^2-2ab\cos(\alpha-\beta)$

$\Rightarrow \sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)=a^2+b^2$

758 **(b)**

$\frac{1}{2}\tan\frac{x}{2}=\frac{1}{2}\cot\frac{x}{2}-\cot x \quad \left[\because \cot x=\frac{1-\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}\right]$

And $\frac{1}{2^2}\tan\frac{x}{2^2}=\frac{1}{2^2}\cot\left(\frac{x}{2^2}\right)-\frac{1}{2}\cot\left(\frac{x}{2}\right)$

Similarly, $\frac{1}{2^3}\tan\left(\frac{x}{2^3}\right)=\frac{1}{2^3}\cot\left(\frac{x}{2^3}\right)-\frac{1}{2^2}\cot\ldots\left(\frac{x}{2^2}\right)$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)=\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right)$

Om adding all the above results, we get

$\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\left(\frac{x}{2^2}\right)+\ldots+\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)=\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x$

759 **(c)**

It is given that

Area of ΔABC = Area of ΔDEF

$\Rightarrow \frac{1}{2}AB\cdot AC\sin A=\frac{1}{2}\ CE\cdot EF\sin E$

$\Rightarrow \sin A=\ \sin E$

$\Rightarrow \sin 2E=\ \sin E$

$\Rightarrow 2E=\pi-E \Rightarrow E=\frac{\pi}{3}\Rightarrow A=2E=\frac{2\pi}{3}$

760 **(c)**

We have,

$\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

$=\ \cos\left(\frac{\pi}{2}-\frac{\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{18}\right)$

$=\ \cos\frac{8\pi}{18}\cos\frac{4\pi}{18}\cos\frac{2\pi}{8}$

$=\ \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}=\frac{\sin(2^3.\pi/9)}{2^3\sin\pi/9}=\frac{1}{2^3}=\frac{1}{8}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	C	C	B	C	A	A	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	D	A	B	A	B	C	C