DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Subject : MATHS
Date :
DPP No. :2

Solutions

Topic :-TRIGONOMETRIC FUNCTIONS

721 **(c)**

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\times\left(1-\cos\right.$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$=\sin^2\frac{\pi}{8}.\sin^2\frac{3\pi}{8}=\frac{1}{4}\left[2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right]^2$$

$$=\frac{1}{4}\left[\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right]^2=\frac{1}{4}\left[\frac{1}{\sqrt{2}}-0\right]^2=\frac{1}{8}$$

722 **(a)**

We have,

$$2\sin\frac{A}{2}=\sqrt{1+\sin A}+\sqrt{1-\sin A}$$

$$\Rightarrow 2\sin\frac{A}{2}-\sqrt{(\cos A/2+\sin A/2)^2}+\sqrt{(\cos A/2-s}$$

$\Rightarrow 2\sin A/2 = |\cos A/2+\sin A/2| + |\sin A2/ - \sin$

$\Rightarrow \cos A/2+\sin A/2 \geq 0$ and $\cos A/2-\sin A/2 \leq 0$

$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4$ and $\pi/4 \leq A \leq 5\pi/4$

$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4$

$\Rightarrow 2n\pi+\pi/4 \leq A/2 \leq 2n\pi+3\pi/4, n\in Z$

723 **(a)**

We have,

$$a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$$

$$\Rightarrow a\left\{\frac{s(s-c)}{ab}\right\}+c\left\{\frac{s(s-a)}{bc}\right\}=\frac{3b}{2}$$

$$\Rightarrow\frac{s}{b}(2s-a-c)=\frac{3b}{2}$$

$\Rightarrow 2s=3b \Rightarrow a+c=2b \Rightarrow a,b,c$ are in A.P.

724 **(a)**

We have,

$$\tan(\theta_1+\theta_2+\ldots+\theta_n)=\frac{S_1-S_3+S_5-S_7+\ldots}{1-S_2+S_4-S_6+\ldots}$$

$$\therefore \tan 5\theta=\frac{{}^5C_1\tan\theta-{}^5C_3\tan^3\theta+{}^5C_5\tan^5\theta}{1-{}^5C_2\tan^2\theta+{}^5C_4\tan^4\theta}$$

725 **(d)**

It is given that a,b,c are in A.P.

$\therefore 2b=a+c$

Now,

$$\frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\left(\tan\frac{A}{2}+\tan\frac{C}{2}\right)\tan\frac{B}{2}$$

$$\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\left\{\frac{\Delta}{s(s-a)}+\frac{\Delta}{s(s-c)}\right\}\frac{\Delta}{s(s-b)}$$

$$\Rightarrow\frac{\tan\frac{A}{B}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\frac{\Delta^2}{s^2(s-b)}\left\{\frac{1}{s-a}+\frac{1}{s-c}\right\}$$

$$\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\frac{\Delta^2 b}{s\,\Delta^2}$$

$$\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}}=\frac{2b}{2s}=\frac{2b}{a+b+c}=\frac{2b}{3b}=\frac{2}{3}\quad[\because a$$

726 **(c)**

We have,

$$2\frac{\cos A}{a}+\frac{\cos B}{b}+2\frac{\cos C}{c}=\frac{a}{bc}+\frac{b}{ac}$$

$$\Rightarrow 2\left(\frac{b^2+c^2-a^2}{2abc}\right)+\frac{c^2+a^2-b^2}{2abc}+2\left(\frac{a^2+b^2-c^2}{2abc}\right)$$

$$\Rightarrow b^2+c^2=a^2\Rightarrow A=\frac{\pi}{2}$$

727 **(d)**

$$2^{n-1}\tan(2^{n-1}\alpha)+2^n\cot(2^n\alpha)$$

$$=2^{n-1}\left[\frac{\sin 2^{n-1}\alpha}{\cos 2^{n-1}\alpha}+2\frac{\cos 2^n\alpha}{\sin 2^n\alpha}\right]$$

$$=2^{n-1}\left[\frac{\cos 2^n\alpha\cos 2^{n-1}\alpha+\sin 2^n\alpha\sin 2^{n-1}\alpha+\cos 2^n\alpha\cos 2^{n-1}\alpha}{\sin 2^n\alpha\cos 2^{n-1}\alpha}\right]$$

$$= 2^{n-1}\left[\frac{\cos 2^{n-1}\alpha(1+\cos 2^n\alpha)}{\sin 2^n\alpha \cos 2^{n-1}\alpha}\right]$$
$$= 2^{n-1}\cot 2^{n-1}\alpha$$
Proceeding in similar way in last, we get
$\tan\alpha + 2\cot 2\alpha$
$$= \frac{\sin\alpha}{\cos\alpha} + 2\frac{\cos 2\alpha}{\sin 2\alpha}$$
$$= \frac{\cos 2\alpha\cos\alpha + \sin 2\alpha\sin\alpha + \cos 2\alpha\cos\alpha}{\sin 2\alpha\cos\alpha}$$
$$= \frac{\cos\alpha(1+\cos 2\alpha)}{2\sin\alpha\cos^2\alpha}$$
$$= \frac{2\cos^2\alpha}{2\sin\alpha}$$
$$= \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$
728 **(c)**
$$\cos^2\left(\frac{\pi}{3}-x\right) - \cos^2\left(\frac{\pi}{3}+x\right)$$
$$= \left[\cos\left(\frac{\pi}{3}-x\right) + \cos\left(\frac{\pi}{3}+x\right)\right]\left[\cos\left(\frac{\pi}{3}-x\right) - \cos\left(\frac{\pi}{3}\right.\right.$$
$$= \left(2\cos\frac{\pi}{3}\cos x\right)\left(2\sin\frac{\pi}{3}\sin x\right)$$
$$= \sin\frac{2\pi}{3}\sin 2x = \frac{\sqrt{3}}{2}\sin 2x$$
Hence, maximum value of given expression is $\frac{\sqrt{3}}{2}$

729 **(d)**
We have,
$$\cos C = \frac{a^2+b^2-c^2}{2\,ab} \Rightarrow \cos C = -1 \Rightarrow C = \pi$$
Which is impossible in a triangle
730 **(c)**
We have,
$$\frac{a}{\cos A} = \frac{b}{\cos B}$$
$\Rightarrow 2R\sin A\cos B = 2R\sin B\cos A$
$\Rightarrow \sin(A-B) = 0 \Rightarrow A = B$
$\therefore 2\sin A\cos B = \sin 2A = \sin(180° - C)$ [$\because 2A +$
$\Rightarrow 2\sin A\cos B = \sin C$
731 **(d)**
Given, $1 + \sin\theta + \sin^2\theta + \ldots\infty = 4 + 2\sqrt{3}$
$$\Rightarrow \frac{1}{1-\sin\theta} = 4 + 2\sqrt{3} \quad [\because 0 < \sin\theta < 1]$$
$$\Rightarrow 1 - \sin\theta = \frac{4-2\sqrt{3}}{16-12} = 1 - \frac{\sqrt{3}}{2}$$
$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$
732 **(c)**
We have,
$a(b^2+c^2)\cos A + b(c^2+a^2)\cos B + c(a^2+b^2)\cos$
$= (ab^2\cos A + ba^2\cos B) + (ac^2\cos A + ca^2\cos C)$
$= ab(b\cos A + a\cos B) + ca(c\cos A + a\cos C) +$
$= abc + abc + abc = 3abc$
733 **(a)**
We have, $\tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$
$$\therefore \sin\theta + \cos\theta = \frac{1}{2}$$
$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$$
$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$
734 **(a)**
We have,
$y = 5x^2 + 2x + 3$
Clearly, it represents an upward opening parabola having its vertex at $(-1/5, 14/5)$
$$\therefore y \geq \frac{14}{5} > 2$$
Now, $y = 2\sin x \leq 2$
Thus, the two curves do not intersect. Hence, there is no common point in the two curves
735 **(d)**
We have,
$(a+b+c)(b+c-a) = \lambda\,bc$
$\Rightarrow 2\,s(2s-a) = \lambda\,bc$
$$\Rightarrow \frac{s(s-a)}{bc} = \frac{\lambda}{4}$$
$$\Rightarrow \cos^2\frac{A}{2} = \frac{\lambda}{4}$$
$$\Rightarrow 0 < \frac{\lambda}{4} < 1 \Rightarrow 0 < \lambda < 4 \quad \left[\because \cos^2\frac{A}{2} \leq 1\right]$$
736 **(c)**
The given expression can be written as
$(1+\cot^2 A)\cot^2 A - (1+\tan^2 A)\tan^2 A - (\cot^2 A$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A$
$- (\cot^2 A - \tan^2 A)(\cot^2 A + \tan^2 A + 1)$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A - (\cot^2 A -$
$= 0$
737 **(b)**
We have, $2b = a + c$
And,

$\Delta = \frac{3}{5} \times \frac{\sqrt{3}}{4}\left(\frac{a+b+c}{3}\right)^2$

$\Rightarrow \Delta = \frac{3\sqrt{3}}{20}b^2$

$\Rightarrow s(s-a)(s-b)(s-c) = \frac{27}{400}b^4$

$\Rightarrow \left(\frac{a+b+c}{2}\right)\left(\frac{b+c-a}{2}\right)\left(\frac{c+a-b}{2}\right)\left(\frac{a+b-c}{2}\right) = \frac{27}{400}b^4$

$\Rightarrow \left(\frac{3b}{2}\right) \times \left(\frac{b+c-2b+c}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2b-c+b-c}{2}\right) = \frac{27}{400}b^4$

[$\because 2b = a+c$]

$\Rightarrow \frac{3b}{2} \times \left(\frac{2c-b}{2}\right) \times \frac{b}{2} \times \left(\frac{3b-2c}{2}\right) = \frac{27}{400}b^4$

$\Rightarrow (2c-b)(3b-2c) = \frac{9b^2}{25}$

$\Rightarrow (6bc - 4c^2 - 3b^2 + 2bc) = \frac{9b^2}{25}$

$\Rightarrow 8bc - 4c^2 - 3b^2 = \frac{9b^2}{25}$

$\Rightarrow \frac{84}{25}b^2 - 8\,bc + 4c^2 = 0$

$\Rightarrow 21b^2 - 50bc + 25c^2 = 0$

$\Rightarrow (7b-5c)(3b-5c) = 0$

$\Rightarrow 7b = 5c$ or, $3b = 5c \Rightarrow \frac{b}{c} = \frac{5}{7}, \frac{5}{3}$

Now,

$2b = a + c \Rightarrow \frac{2b}{c} = \frac{a}{c} + 1 \Rightarrow \frac{a}{c} = \frac{3}{7}, \frac{7}{3}$

Hence, $a:b:c = 3:5:7$

738 **(a)**

$$\sqrt{\frac{a+b}{a-b}}-\sqrt{\frac{a-b}{a+b}}$$

$$=\sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}}-\sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$$

$$=\sqrt{\frac{1+\tan\alpha}{1-\tan\alpha}}-\sqrt{\frac{1-\tan\alpha}{1+\tan\alpha}}$$

$$=\frac{(1+\tan\alpha)-(1-\tan\alpha)}{\sqrt{1-\tan^2\alpha}}$$

$$=\frac{2\tan\alpha}{\sqrt{1-\tan^2\alpha}}=\frac{2\sin\alpha}{\sqrt{\cos 2\alpha}}$$

739 **(b)**

Since, $\sin\theta+\cos\theta=x$ ...(i)

and $\sin^6\theta+\cos^6\theta=\frac{1}{4}[4-3(x^2-1)^2]$

On equation Eq (i), we get

$\sin 2\theta = x^2-1\le 1 \quad (\because \sin 2\theta \le 1)$

$\Rightarrow x^2\le 2 \Rightarrow -\sqrt{2}\le x\le\sqrt{2}$

Now, $\sin^6\theta+\cos^6\theta = (\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$$=1-3\sin^2\theta\cos^2\theta=1-\frac{3}{4}\sin^2 2\theta$$

$$=1-\frac{3}{4}(x^2-)^2=\frac{1}{4}[4-3(x^2-1)^2]$$

Thus, the given result will hold true only when $x^2\le 2$ and not for all real values of x

740 **(b)**

We have,

$$\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$$

$\Rightarrow \sin(B+C)\sin(B-C) = \sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2 B-\sin^2 C = \sin^2 A-\sin^2 B$

$\Rightarrow b^2-c^2=a^2-b^2 \Rightarrow a^2, b^2, c^2$ are in A.P.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	A	D	C	D	C	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	A	A	D	C	B	A	B	B