DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :10

Topic :-TRIGONOMETRIC FUNCTIONS

881 **(b)**

Let $f(x) = \sin x \cos x = \frac{1}{2}\sin 2x$

We know that, $-1 \le \sin 2x \le 1$

$$\Rightarrow -\frac{1}{2} \le \frac{1}{2}\sin 2x \le \frac{1}{2}$$

Thus, the greatest and least value of $f(x)$ are $\frac{1}{2}$ and $\frac{1}{2}$ respectively

882 **(b)**

We have,

$x^2 + 4xy + y^2$

$= (X\cos\theta - Y\sin\theta)^2 + 4(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + (X\sin\theta + Y\cos\theta)^2$

$= (1 + 4\sin\theta\cos\theta)X^2 + 4(\cos^2\theta - \sin^2\theta)XY + (1 - 4\sin\theta\cos\theta)Y^2$

$\therefore x^2 + 4xy + y^2 = AX^2 + BY^2$

$\Rightarrow (1 + 2\sin 2\theta)X^2 + 4\cos 2\theta\, XY + (1 - 2\sin 2\theta)Y^2$

$= AX^2 + BY^2$

$\Rightarrow \cos 2\theta = 0, A = 1 + 2\sin 2\theta, B = 1 - 2\sin 2\theta$

$\Rightarrow \theta = \frac{\pi}{4}$ and $A = 1 + 2 = 3, B = 1 - 2 = -1$

883 **(d)**

Given, $3\cos 2x - 10\cos x + 7 = 0$

$\Rightarrow 6\cos^2 x - 10\cos x + 4 = 0$

$[\because \cos 2x = 2\cos^2 x - 1]$

$\Rightarrow 2(3\cos x - 2)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = \frac{2}{3}$

Since, $\cos x$ is positive in Ist and IIIrd quadrant.

Hence, total number of solutions are 4

884 **(a)**

$\cos\alpha\sin(\beta - \gamma) + \cos\beta\sin(\gamma - \alpha) + \cos\gamma\sin(\alpha - \beta)$

$=\cos\alpha\,[\sin\beta\cos\gamma-\cos\beta\sin\gamma]+\cos\beta[\sin\gamma\cos\alpha-\cos\gamma\sin\alpha]+$

$\cos\gamma[\sin\alpha\cos\beta-\cos\alpha\sin\beta]$

$=0$

885 **(d)**

Given, $A+B=45°$

$\Rightarrow \cot(A+B)=1$

$\Rightarrow \dfrac{\cot A\cot B-1}{\cot A+\cot B}=1$

$\Rightarrow \cot A\cot B-(\cot A+\cot B)=1 \quad ...(i)$

Now, $(\cot A-1)(\cot B-1)=\cot A\cot B-(\cot A+\cot B)+1$

$=1+1=2$ [from Eq. (i)]

886 **(c)**

Let $I=[\sin x+\cos x]^{1+\sin 2x}$

$=\left[\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)\right]^{1+\sin 2x}$

At $x=\frac{\pi}{4}$,

$I=\left[\sqrt{2}\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)\right]^{1+\sin\frac{2\pi}{4}}$

$=(\sqrt{2})^2=2$

887 **(d)**

The given expression can be written as

$\cos^6 x(\cos^6 x+3\cos^4 x+3\cos^2 x+1)+2\cos^4 x+\cos^2 x-2$

$=\sin^3 x(\cos^2 x+1)^3+2\cos^4 x+\cos^2 x-2$

$=\sin^3 x(\sin x+1)^3+2\sin^2 x+\cos^2 x-2$

$[\because \sin x+\sin^2 x=1\Rightarrow \sin x=\cos^2 x]$

$=(\sin x+\sin^2 x)^3+\sin^2 x+(\sin^2 x+\cos^2 x)-2$

$=1^3+\sin^2 x+1-2=\sin^2 x$

888 **(b)**

$\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

$=\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{4}(3) + \frac{1}{4}(3) = \frac{3}{2}$

889 **(c)**

Given, $\frac{\sin(x+3\alpha)}{\sin(\alpha - x)} = 3$

Applying componendo and dividendo, we get

$$\frac{\sin(x+3\alpha) + \sin(\alpha - x)}{\sin(x+3\alpha) - \sin(\alpha - x)} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{2\sin 2\alpha \cos(\alpha + x)}{2\cos 2\alpha \sin(\alpha + x)} = 2$$

$$\Rightarrow \frac{\tan 2\alpha}{\tan(\alpha + x)} = 2$$

$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} \times \frac{(1-\tan\alpha\tan x)}{(\tan\alpha + \tan x)} = 2$$

$\Rightarrow \tan\alpha - \tan^2\alpha\tan x = \tan\alpha + \tan x - \tan^3\alpha - \tan^2\alpha\tan x$

$\Rightarrow \tan x = \tan^3\alpha$

890 **(a)**

We have,

$\cos\alpha\sin(\beta - \gamma) + \cos\beta\sin(\gamma - \beta) + \cos\gamma\sin(\alpha - \beta)$

$$= \frac{1}{2}\{\sin(\alpha + \beta - \gamma) + \sin(\beta - \gamma - \alpha) + \sin(\gamma - \alpha + \beta) + \sin(\gamma - \alpha - \beta) + \sin(\alpha - \beta + \gamma) + \sin(\alpha - \beta - \gamma)\}$$

$$= \frac{1}{2}\{\sin(\alpha + \beta - \gamma) - \sin(\alpha - \beta + \gamma) - \sin(\alpha - \beta - \gamma) - \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma) + \sin(\alpha - \beta - \gamma)\}$$

$$= \frac{1}{2} \times 0 = 0$$

891 **(c)**

$\sin A + \cos A = m$ [given]

$\Rightarrow \sin^3 A + \cos^3 A + 3\cos A\sin A$

$(\sin A + \cos A) = m^3$

$\Rightarrow n + 3m\sin A\cos A = m^3$ ....(i)

$[\because \sin^3 A + \cos^3 A = n]$

Again, $\sin A + \cos A = m$

$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A\cos A = m^2$

$\Rightarrow \sin A\cos A = \frac{m^2 - 1}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$$n + 3m\frac{(m^2-1)}{2} = m^3$$

$\Rightarrow 2n + 3m^3 - 3m = 2m^3$

$\Rightarrow m^3 - 3m + 2n = 0$

892 **(b)**

We have,

$(\sec\theta - 1) = (\sqrt{2} - 1)\tan\theta$

$\Rightarrow 1 - \cos\theta = (\sqrt{2} - 1)\sin\theta$

$\Rightarrow 2\sin^2\frac{\theta}{2} = 2(\sqrt{2} - 1)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\Rightarrow \sin\frac{\theta}{2} = 0 \text{ or}, \tan\frac{\theta}{2} = \sqrt{2} - 1 = \tan\frac{\pi}{8}$

$\Rightarrow \frac{\theta}{2} = n\,\pi \text{ or}, \frac{\theta}{2} = n\,\pi + \frac{\pi}{8}, n \in Z$

$\Rightarrow \theta = 2\,n\,\pi, \theta = 2\,n\,\pi + \frac{\pi}{4}, n \in Z$

893 **(d)**

Given, $\cos\theta + \sin 2\theta = 0$

$\Rightarrow \cos\theta + 2\sin\theta\cos\theta = 0$

$\Rightarrow \cos\theta(1 + 2\sin\theta) = 0$

$\Rightarrow \cos\theta = 0 \text{ or } \sin\theta = -\frac{1}{2}$

For $\theta \in [-\pi, \pi]$

$\theta = \frac{\pi}{2}, -\frac{\pi}{2}$

Or $\theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$

894 **(b)**

We have,

$\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \frac{\pi}{2}\sin\theta = r\,\pi + \frac{\pi}{2} - \frac{\pi}{2}\cos\theta,\ r \in Z$

$\Rightarrow \sin\theta + \cos\theta = (2r + 1), r \in Z$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{2r + 1}{\sqrt{2}}, r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2r + 1}{\sqrt{2}}, r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or} -\frac{1}{\sqrt{2}}$ [For $r = 0, -1$]

$\Rightarrow \theta - \frac{\pi}{4} = 2\,r\,\pi \pm \frac{\pi}{4}, r \in Z$

$\Rightarrow \theta = 2\,r\,\pi \pm \frac{\pi}{4} + \frac{\pi}{4}, r \in Z$

$\Rightarrow \theta = 2\,r\,\pi, 2\,r\,\pi + \frac{\pi}{2}, r \in Z$

But, $\theta = 2\,r\,\pi + \frac{\pi}{2}, r \in Z$ gives extraneous roots as it does not satisfy the given equation. Therefore,

$$\theta = 2\,r\,\pi, r \in Z$$

895 **(b)**

$\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2$

$\Rightarrow \tan\theta + \tan\left(\frac{\pi}{2} + \left(\frac{\pi}{4} + \theta\right)\right) = 2$

$\Rightarrow \tan\theta - \cot\left(\frac{\pi}{4} + \theta\right) = 2$

$\Rightarrow \tan\theta - \frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\frac{\pi}{4} + \cot\theta} = 2$

$\Rightarrow \tan\theta - \frac{\cot\theta - 1}{1 + \cot\theta} = 2$

$\Rightarrow \tan\theta - \frac{1 - \tan\theta}{1 + \tan\theta} = 2$

$\Rightarrow \tan\theta + \tan^2\theta - 1 + \tan\theta = 2 + 2\tan\theta$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3} = \pm\tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in Z$

896 **(b)**

We have,

$(1 + \tan\theta)(1 + \tan\phi) = 2$

$\Rightarrow 1 + \tan\theta + \tan\phi + \tan\theta\tan\phi = 2$

$\Rightarrow \tan\theta + \tan\phi = 1 - \tan\theta\tan\phi$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta + \phi) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4}, n \in Z$

897 **(c)**

Given, $2\sec 2\alpha = \tan\beta + \cot\beta$

$\Rightarrow 2\sec 2\alpha = \frac{\sin^2\beta + \cos^2\beta}{\sin\beta\cos\beta}$

$\Rightarrow \frac{2}{\cos 2\alpha} = \frac{1}{\sin\beta\cos\beta}$

$\Rightarrow \sin 2\beta = \cos 2\alpha$

$\Rightarrow \alpha + \beta = \frac{\pi}{4}$

898 **(a)**

We have,

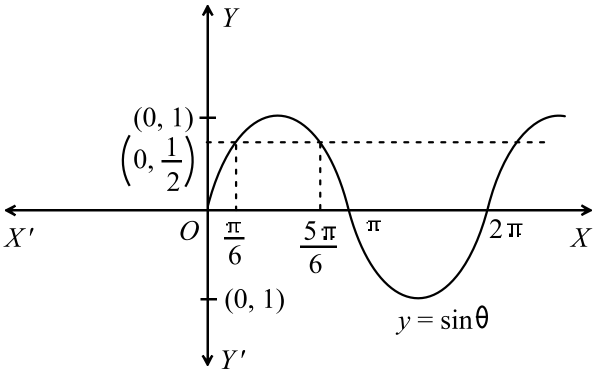
$2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow 2\sin\theta - 1 < 0 \quad [\because -1 \le \sin\theta \le 1 \ \therefore \sin\theta - 2 < 0]$

$\Rightarrow \sin\theta < \frac{1}{2}$

$\Rightarrow \theta \in (0, \pi/6) \cup (5\pi/6, \pi)$



899 **(d)**

Given that,

$\tan A - \tan B = x$ ...(i)

and $\cot B - \cot A = y$ ...(ii)

Now, $\cot(A - B) = \frac{1}{\tan(A - B)}$

$= \dfrac{1 + \tan A \tan B}{\tan A - \tan B}$

$= \dfrac{1}{\tan A - \tan B} + \dfrac{\tan A \tan B}{\tan A - \tan B}$

$= \frac{1}{x} + \frac{1}{y}$ [from Eqs.(i)and(ii)]

900 **(b)**

We have,

$\dfrac{a \cos B - b \cos A}{a - b}$

$= \dfrac{a\left(\frac{c^2 + a^2 - b^2}{2ac}\right) - b\left(\frac{b^2 + c^2 - a^2}{2bc}\right)}{a - b}$

$= \dfrac{2(a^2 - b^2)}{2c(a - b)} = \dfrac{a + b}{c} = \dfrac{2c}{c} = 2 \left[\begin{array}{c} \because a, c, b \text{ are in A.P.} \\ \therefore a + b = 2c \end{array}\right]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10

A.	B	B	D	A	D	C	D	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	B	B	C	A	D	B