DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :1

Topic :-TRIGONOMETRIC FUNCTIONS

702 **(c)**

We have,

$\sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x$

Now,

$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$

$= \cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1$

$= \cos^6 x(\cos^2 x + 1)^3 - 1$

$= \sin^3 x(\sin x + 1)^3 - 1$

$= (\sin^2 x + \sin x)^3 - 1$

$= (\sin^2 x + \cos^2 x)^3 - 1 \quad [\because \sin x = \cos^2 x]$

$= 1 - 1 = 0$

703 **(c)**

Let $a = 3x + 4y, b = 4x + 3y$ and $c = 5x + 5y$.

Clearly, c is the largest side and thus the largest angle C is given by

$$\cos C \frac{a^2 + b^2 - c^2}{2ab} = \frac{-2xy}{2(12x^2 + 25xy + 12y^2)} < 0$$

$\Rightarrow C$ is an obtuse angle

704 **(a)**

Let $a = x^2 + x + 1, b = x^2 - 1$ and $c = 2x + 1$. Then,

$a - b = x + 2 > 0 \quad [\because x > 1]$

$a - c = x^2 - x > 0 \quad [\because x > 1]$

So, a is the largest side

Hence, the largest angle is given by

$$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos\theta = \frac{(x^2-1)^2 + (2x+1)^2 - (x^2+x+1)^2}{2(x^2-1)(2x+1)} =$$

$\Rightarrow \theta = 2\pi/3 = 120°$

705 **(c)**

We have,

$$\frac{1}{2}\,a\,p_1 = \Delta, \frac{1}{2}\,b\,p_2 = \Delta, \frac{1}{2}\,c\,p_3 = \Delta$$

$$\Rightarrow p_1 = \frac{2\,\Delta}{a}, p_2 = \frac{2\,\Delta}{b}, p_3 = \frac{2\,\Delta}{c}$$

$$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2+b^2+c^2}{4\,\Delta^2}$$

$$\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{a}{2\,\Delta} + \frac{b}{2\,\Delta} - \frac{c}{2\,\Delta} = \frac{a+b-c}{2\,\Delta} = \frac{2(s\cdot}{2}$$

706 **(c)**

We have,

$$\cos C = \frac{63}{65} \Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{63}{65} \Rightarrow \frac{26^2+30^2-c^2}{2\times 26\times 30} =$$

$$\Rightarrow 676 + 900 - c^2 = 1260 \Rightarrow c^2 = 64 \Rightarrow c = 8$$

Thus, we have

$a = 26, b = 30$ and $c = 8$

$$\therefore 2s = a + b + c \Rightarrow 2s = 26 + 30 + 8 = 64 \Rightarrow s = 32$$

Also, $\Delta = \sqrt{s(s-a)(s-b)(s-c)} =$

$$\sqrt{32\times 6\times 2\times 24} = 96$$

Hence, $r_2 = \frac{\Delta}{s-b} = \frac{96}{32-30} = 48$

707 **(c)**

$$\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 100^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots 0 \ldots \cos 100^\circ = 0$$

708 **(b)**

We have,

$$\sin\frac{\pi}{2} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{2\sin^2\frac{\pi}{7} + 2\sin\frac{\pi}{7}\sin\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\right.$$

$$= \frac{1}{2\sin\left(\frac{\pi}{7}\right)}\left\{1 - \cos\frac{2\pi}{7} + \cos\frac{\pi}{7} - \cos\frac{3\pi}{7} + \cos\frac{2\pi}{7}\right.$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{1 + \cos\frac{\pi}{7}\right\} = \frac{2\cos^2\frac{\pi}{14}}{4\sin\frac{\pi}{14}\cos\frac{\pi}{14}} = \frac{1}{2}\cot\frac{\pi}{14}$$

709 **(a)**

Let $f(x) = \sqrt{3}\cos x + \sin x$

$\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) = 2\sin\left(x + \frac{\pi}{3}\right)$

Since, $-1 \le \sin\left(x + \frac{\pi}{3}\right) \le 1$

Hence, $f(x)$is maximum, if $x + \frac{\pi}{3} = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{6} = 30°$

710 **(b)**

$\sin^2 17.5° + \sin^2 72.5°$

$= \sin^2 17.5° + \cos^2 17.5°$ $[\because$ $\sin(9\theta - \theta) =$ cos

$= 1 = \tan^2 45°$

711 **(a)**

We have,

$a \sin A = b \sin B$

$\Rightarrow a \cdot ak = b \cdot bk \Rightarrow a = b \Rightarrow \Delta ABC$ is isosceles

712 **(b)**

We know that $\sin^2\theta \ge 1$

$\Rightarrow \frac{4xy}{(x+y)^2} \ge 1$

$\Rightarrow 4xy \ge (x+y)^2$

$\Rightarrow (x-y)^2 \le 0$

$\Rightarrow x - y = 0 \Rightarrow y = x$

And $x \ne 0, \ y \ne 0$

713 **(b)**

Given that, $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$

$\Rightarrow x + \frac{1}{x} = 2\cos\theta$ ....(i)

We know that, $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (2\cos\theta)^2 - 2 = 4\cos^2\theta - 2$

$= 2\cos 2\theta$ [from Eq.(i)]

$\therefore \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = \frac{1}{2} \times 2\cos 2\theta = \cos 2\theta$

714 **(d)**

$\text{sech}^{-1}(\sin\theta)$

$= \cosh^{-1}(\text{cosec}\,\theta)$

$$= \log\left[\operatorname{cosec}\theta + \sqrt{(\operatorname{cosec}^2\theta - 1)}\right]$$

$$= \log\left[\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right] = \log\cot\frac{\theta}{2}$$

715 **(d)**

Consider the curves $y = 2^{\cos x}$ and $y = |\sin x|$.
Clearly, both the curves are symmetrical about y-axis as $\cos x$ and $|\sin x|$ are even functions
Also, $y = 2^{\cos x}$ and $y = |\sin x|$ intersect at two points in $[0, 2\pi]$
Hence, there are four solutions of the given equation

716 **(d)**

We have,

$\cos(\lambda\sin\theta) = \sin(\lambda\cos\theta)$

$\Rightarrow \cos(\lambda\sin\theta) = \cos\left(\frac{\pi}{2} - \lambda\cos\theta\right)$

$\Rightarrow \lambda\sin\theta = \frac{\pi}{2} - \lambda\cos\theta \Rightarrow \cos\theta + \sin\theta = \frac{\pi}{2\lambda}$

This equation will have a solution if

$\left|\frac{\pi}{2\lambda} \le \sqrt{2}\right|$ $\quad\left[\because |a\cos\theta + b\sin\theta| \le \sqrt{a^2 + b^2}\right]$

$\Rightarrow \frac{\pi}{2\lambda} \le \sqrt{2} \Rightarrow \lambda \ge \frac{\pi}{2\sqrt{2}}$ $\quad[\because \lambda > 0]$

717 **(c)**

We have,

$c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$

$\therefore c_1 - c_2 = \sqrt{(c_1 + c_2)^2 - 4c_1c_2}$

$\Rightarrow c_1 - c_2 = \sqrt{4b^2\cos^2 A - 4(b^2 - a^2)} = 2\sqrt{a^2 - b^2 s}$

718 **(b)**

We have,

$\tan\alpha = (1 + 2^{-x})^{-1} = \frac{2^x}{2^x + 1}$ and $\tan\beta = \frac{1}{2^{x+1} +}$

$\therefore \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\Rightarrow \tan(\alpha + \beta) = \frac{2^x(2^{x+1} + 1) + (2^x + 1)}{(2^x + 1)(2^{x+1} + 1) - 2^x}$

$\Rightarrow \tan(\alpha + \beta) = \frac{2(2^x)^2 + 2.2^x + 1}{2(2^x)^2 + 2.2^x + 1} = 1 \Rightarrow \alpha + \beta = \pi,$

719 **(a)**

Given, $f(x) = \sin x(1 + \cos x)$

It is minimum at $x = \frac{\pi}{3}$

$$\therefore f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}\left(1 + \frac{1}{2}\right) = \frac{3\sqrt{3}}{4}$$

720 **(c)**

We have,

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11}\cos\frac{9\pi}{11}$$

$$= \frac{\cos\left\{\frac{\pi}{11} + \left(\frac{5-1}{2}\right)\frac{2\pi}{11}\right\}\sin\left(\frac{5\pi}{11}\right)}{\sin\left(\frac{\pi}{11}\right)}$$

$$= \frac{\cos\frac{5\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \frac{1}{2}\frac{\sin\left(\frac{10\pi}{11}\right)}{\sin\frac{\pi}{11}} = \frac{1}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	C	A	C	C	C	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	B	D	D	D	C	B	A	C