

Topic :-STRAIGHT LINES

361 (a)

Let the slope of first line be m , then slope of second line is $5m$.

Then, $m + 5m = -\frac{2h}{b}$ and $m \cdot 5m = \frac{a}{b}$

$$\Rightarrow m = -\frac{2h}{6b} = \frac{-h}{3b}$$

$$\therefore 5m^2 = \frac{a}{b} \Rightarrow 5\left(-\frac{h}{3b}\right)^2 = \frac{a}{b}$$

$$\Rightarrow \frac{5h^2}{9b^2} = \frac{a}{b} \Rightarrow 5h^2 = 9ab$$

362 (a)

We have,

$$|x| = |y| \Rightarrow x = \pm y \Rightarrow x + y = 0, x - y = 0$$

Let $(t, 4 - t)$ be the required point. It is equidistant from the lines $|x| = |y|$

$$\therefore \left| \frac{t + 4 - t}{\sqrt{2}} \right| = \left| \frac{t - (4 - t)}{\sqrt{2}} \right|$$

$$\Rightarrow 4 = |2t - 4| \Rightarrow t - 2 = \pm 2 \Rightarrow t = 0, 4$$

Hence, required points are $(0, 4)$ and $(4, 0)$

363 (d)

$$\text{Equation of line is } \frac{x}{3} + \frac{y}{4} = 1$$

$$\Rightarrow 4x + 3y - 12 = 0$$

$$\text{Now, distance from origin} = \left| \frac{4 \times 0 + 3 \times 0 - 12}{\sqrt{3^2 + 4^2}} \right| = \frac{12}{5} \text{ units}$$

364 (c)

$$\text{As } m \in \left(\frac{1}{2}, 3\right)$$

\therefore Line $y = mx + 4$ lies between

$$y = 3x + 1 \text{ and } 2y = x + 3$$

$$\text{Slope of given lines are } m_2 = 3, m = m \text{ and } m_1 = \frac{1}{2}$$

$$\therefore \tan \theta = \frac{3 - m}{1 + 3m}$$

$$\text{and } \tan \theta = \frac{m - \frac{1}{2}}{1 + \frac{m}{2}}$$

$$\Rightarrow \frac{3 - m}{1 + 3m} = \frac{2m - 1}{2 + m}$$

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$\therefore m = \frac{2 \pm \sqrt{4 + 196}}{2 \times 7} = \frac{1}{7}(1 \pm 5\sqrt{2})$$

365 (d)

The point of intersection of the given lines is

$$\left(\frac{ab}{a+b}, \frac{ab}{a+b} \right)$$

Clearly, it satisfies equation of options (a),(b) and (c)

366 (c)

Equation of the straight lines are

$$3x - 4y + 7 = 0 \dots(i)$$

$$\text{and } 12x + 5y - 2 = 0 \dots(ii)$$

The equation of bisectors of the angles between these lines are

$$\frac{3x - 4y + 7}{\sqrt{3^2 + 4^2}} = \frac{12x + 5y - 2}{\sqrt{12^2 + 5^2}}$$

$$\Rightarrow \frac{3x - 4y + 7}{5} = \frac{12x + 5y - 2}{13}$$

$$\Rightarrow 39x - 52y + 91 = 60x + 25y - 10$$

$$\Rightarrow 21x + 77y - 101 = 0$$

367 (b)

Given equation of pair of lines can be written as

$$(3x - y)(x + 2y) = 0$$

Slope of separate equations of line $3x - y = 0$ is 3 and $x + 2y = 0$ is $-\frac{1}{2}$

$$\text{Thus, required sum} = 3 - \frac{1}{2} = \frac{5}{2}$$

Alternate

Sum of slope of the lines $3x^2 + 5xy - 2y^2 = 0$ is

$$m_1 + m_2 = -\frac{h}{b} = \frac{5}{2}$$

368 (b)

Let the another equation of line is

$$x - 2y + 1 = 0$$

\therefore Equation of bisector of angle between two lines is

$$\frac{2x - y - 1}{\sqrt{4 + 1}} = \pm \frac{x - 2y + 1}{\sqrt{1 + 4}}$$

$$\Rightarrow x + y - 2 = 0 \text{ and } x = y$$

369 (d)

Given equation can be rewritten as

$$a(x + y - 1) + b(2x - 3y + 1) = 0$$

This is the form of intersection of two lines.

$$\therefore x + y - 1 = 0 \dots(i)$$

$$\text{and } 2x - 3y + 1 = 0 \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = \frac{2}{5} \text{ and } y = \frac{3}{5}$$

Hence, coordinates of required point are $(\frac{2}{5}, \frac{3}{5})$

370 (a)

Since, $ax + by + c = 0$ is always passes through $(1, -2)$

$$\therefore a - 2b + c = 0$$

$$\Rightarrow 2b = a + c$$

Therefore, a, b and c are in AP

371 (a)

Let the locus of point be (x, y)

Area of triangle with points (x, y) , $(1, 5)$ and $(3, -7)$ is 21 sq unit

$$\therefore \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 1 & 5 & 1 \\ 3 & -7 & 1 \end{vmatrix} = 21$$

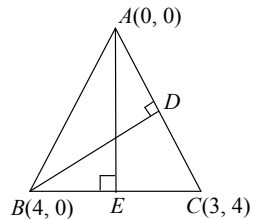
$$\Rightarrow \frac{1}{2} [x(5 + 7) - y(1 - 3) + 1(-7 - 15)] = 21$$

$$\Rightarrow \frac{1}{2} [12x + 2y - 22] = 21$$

$$\Rightarrow 6x + y - 32 = 0$$

372 (c)

Now, we take $BD \perp AC$ and $AE \perp BC$



$$\text{Slope of } BD = -\frac{3}{4}$$

$$\text{Equation of } BD, y - 0 = \frac{-3}{4}(x - 4)$$

$$\Rightarrow 4y = -3x + 12$$

$$\Rightarrow 3x + 4y - 12 = 0 \dots(i)$$

$$\text{and slope of } AE = \frac{1}{4}$$

$$\therefore \text{Equation of } AE, y - 0 = \frac{1}{4}(x - 0)$$

$$\Rightarrow x - 4y = 0 \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = 3, \quad y = \frac{3}{4}$$

∴ Orthocentre of the triangle is $\left(3, \frac{3}{4}\right)$

373 (b)

Let $A(2, -1)$ be one vertex of an equilateral triangle ABC . Then, its altitude is the length of the perpendicular from $A(2, -1)$ on $c + y - 2 = 0$ i.e.

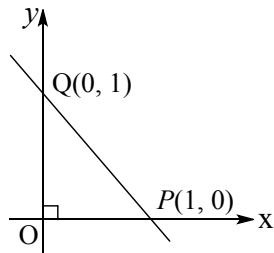
$$AD = \left| \frac{2 - 1 - 2}{\sqrt{1 + 1}} \right| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2}(\text{Side}) = \frac{1}{\sqrt{2}} \Rightarrow \text{side} = \sqrt{\frac{2}{3}}$$

374 (a)

We have, $x + y = 1$... (i)

and $xy = 0$... (ii)



On putting $x = 1 - y$ from Eq. (i) into Eq. (ii), we get

$$(1 - y)y = 0$$

$$\Rightarrow y = 0, 1$$

$$\text{At } y = 0 \Rightarrow x = 1$$

$$\text{and at } y = 1 \Rightarrow x = 0$$

∴ Coordinates of the vertices of a triangle are $(0, 0)$, $(1, 0)$ and $(0, 1)$

∴ Point $(0, 0)$ is its orthocentre

375 (a)

The equation of required line is

$$3x^2 + 4xy - 4x(2x + y) + (2x + y)^2 = 0$$

$$\Rightarrow 3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy = 0$$

$$\Rightarrow -x^2 + y^2 + 4xy = 0$$

$$(\text{Coefficient of } x^2) + (\text{Coefficient of } y^2) = -1 + 1 = 0$$

∴ Lines are mutually perpendicular.

i.e., Angle between lines is $\frac{\pi}{2}$.

376 (a)

The equation of given line is

$$y = mx + \frac{a}{m} \dots (i)$$

The equation of line perpendicular to Eq. (i) is

$$my + x + \lambda = 0 \dots (ii)$$

This line passing through $(a, 0)$.

$$0 + a + \lambda = 0 \Rightarrow \lambda = -a$$

On putting this value on λ in Eq. (ii) and solving with Eq. (i), we get

$$x = 0 \text{ and } y = \frac{a}{m}$$

Coordinates of the foot of perpendicular are $(0, \frac{a}{m})$.

377 (b)

$$\therefore \text{Slope of perpendicular} = - \left[\frac{\cos \alpha - \cos \beta}{\sin \alpha - \sin \beta} \right] = \tan \frac{\alpha + \beta}{2}$$

\therefore Equation of perpendicular is

$$y = \tan \left(\frac{\alpha + \beta}{2} \right) x \quad \dots(i)$$

On solving the Eq. (i) with the line, we get

$$\left[\frac{a}{2}(\cos \alpha + \cos \beta), \frac{a}{2}(\sin \alpha + \sin \beta) \right]$$

378 (d)

Mid point of the line joining the points $(4, -5)$ and $(-2, 9)$ is

$$\left(\frac{4-2}{2}, \frac{-5+9}{2} \right) \text{ ie, } (1, 2)$$

\therefore Inclination of straight line passing through point $(-3, 6)$ and mid point $(1, 2)$ is

$$m = \frac{2-6}{1+3} = \frac{-4}{4} = -1$$

$$\therefore \tan \theta = -1 \Rightarrow \theta = \frac{3\pi}{4}$$

379 (d)

Given pair of line is

$$x^2 \sin^2 \alpha + y^2 \sin^2 \alpha = x^2 \cos^2 \theta + y^2 \sin^2 \theta - 2xy \sin \theta \cos \theta$$

$$\Rightarrow x^2(\sin^2 \alpha - \cos^2 \theta) + y^2(\sin^2 \alpha - \sin^2 \theta) + 2(\sin \theta \cos \theta)xy = 0$$

On comparing with $ax^2 + by^2 + 2hxy = 0$

We get, $a = \sin^2 \alpha - \cos^2 \theta$,

$b = \sin^2 \alpha - \sin^2 \theta$ and $h = \sin \theta \cos \theta$

Let θ be the angle between the pair of lines.

$$\therefore \tan \theta = \left| \frac{2\sqrt{\sin^2 \theta \cos^2 \theta - (\sin^2 \alpha - \cos^2 \theta) \times (\sin^2 \alpha - \sin^2 \theta)}}{\sin^2 \alpha - \cos^2 \theta + \sin^2 \alpha - \sin^2 \theta} \right|$$

$$= \left| \frac{2\sqrt{\sin^2 \theta \cos^2 \theta - (\sin^2 \alpha)^2 + \sin^2 \alpha \sin^2 \theta + \sin^2 \alpha \cos^2 \theta - \sin^2 \theta \cos^2 \theta}}{- (1 - 2\sin^2 \alpha)} \right|$$

$$= \left| \frac{2\sqrt{\sin^2 \alpha (\sin^2 \theta + \cos^2 \theta) - (\sin^2 \alpha)^2}}{- \cos 2 \alpha} \right|$$

$$= \left| \frac{2\sqrt{\sin^2 \alpha (1 - \sin^2 \alpha)}}{- \cos 2 \alpha} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\sin 2 \alpha}{\cos 2 \alpha} \right| = \tan 2 \alpha$$

$$\Rightarrow \theta = 2\alpha$$

380 (b)

Given equations of line and circle are respectively

$$\sqrt{3}x + y = 2 \dots(i)$$

$$\text{and } x^2 + y^2 = 4 \dots(ii)$$

From Eqs. (i) and (ii), we get

$$x^2 + (2 - \sqrt{3}x)^2 = 4$$

$$\Rightarrow 4x^2 - 4\sqrt{3}x = 0$$

$$\Rightarrow x(x - \sqrt{3}) = 0 \Rightarrow x = 0, \sqrt{3}$$

\therefore Points of intersection of line and circle are $(0, 2)$ and $(\sqrt{3}, -1)$.

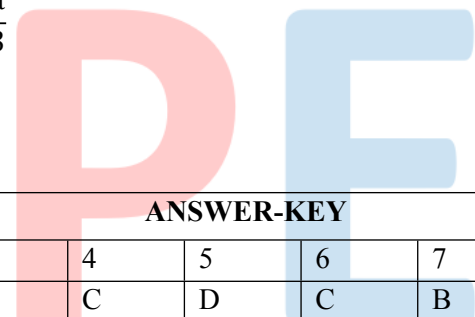
Slope, of line joining $(0, 0)$ and $(0, 2)$

$$= \frac{2 - 0}{0 - 0} = \infty \Rightarrow \theta_1 = \frac{\pi}{2}$$

Also, slope of line joining $(0, 0)$ and $(\sqrt{3}, -1)$

$$= \frac{-1}{\sqrt{3}} \Rightarrow \theta_2 = \frac{\pi}{6}$$

$$\therefore \text{Required angle} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$



ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	C	D	C	B	B	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	A	A	B	D	D	B