

## Topic :-STRAIGHT LINES

341 (b)

The equation of a line passing through  $P(1,1)$  and parallel to  $2x - y = 0$  is

$$\frac{x-1}{\cos \theta} = \frac{y-1}{\sin \theta}, \text{ where } \tan \theta = 2$$

$$\text{i.e. } \frac{x-1}{1/\sqrt{5}} = \frac{y-1}{2/\sqrt{5}}$$

Since  $P$  is translated in the first quadrant through a unit distance, therefore the coordinates of  $P$  are given by

$$\frac{x-1}{1/\sqrt{5}} = \frac{y-1}{2/\sqrt{5}} = \pm 1$$

$$\Rightarrow x = 1 \pm \frac{1}{\sqrt{5}}, y = 1 \pm \frac{2}{\sqrt{5}}$$

Hence, the coordinates of  $P$  are  $\left(1 \pm \frac{1}{\sqrt{5}}, 1 \pm \frac{2}{\sqrt{5}}\right)$

342 (a)

$$\text{Given, } \frac{1}{a}x^2 + \frac{1}{b}y^2 + 2\frac{1}{h}xy = 0$$

$$\therefore m_1 + m_2 = -\frac{\frac{2}{h}}{\frac{1}{b}} = \frac{-2b}{h} \dots(i)$$

$$\text{and } m_1 m_2 = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{b}{a} \dots(ii)$$

Also given  $m_2 = 2m_1$

$$\Rightarrow 3m_1 = \frac{-2b}{h} \text{ [from Eq.(i)]} \dots(iii)$$

$$\text{and } 2m_1^2 = \frac{b}{a} \text{ [from Eq.(ii)]} \dots(iv)$$

From Eqs. (iii) and (iv),

$$\frac{9m_1^2}{2m_1^2} = \frac{4b^2}{h^2} \times \frac{a}{b}$$

$$\Rightarrow \frac{9}{8} = \frac{ba}{h^2} \text{ or } ab:h^2 = 9:8$$

344 (a)

Clearly the point (3,0) does not lie on the diagonal  $x = 2y$ . Let  $m$  be the slope of a side passing through (3,0). Then, its equation is

$$y - 0 = m(x - 3) \dots(i)$$

Since the angle between a diagonal and a side of a square is  $\pi/4$ . Therefore, angle between  $x = 2y$  and  $y - 0 = m(x - 3)$  is also  $\pi/4$ . Consequently, we have

$$\tan \frac{\pi}{4} = \pm \frac{m - 1/2}{1 + m/2} \Rightarrow m = 3, -\frac{1}{3}$$

Substituting the values of  $m$  in (i), we obtain

$$y - 3x + 9 = 0 \text{ and } 3y + x - 3 = 0 \text{ as the required sides}$$

345 (a)

Any line which is perpendicular to  $\sqrt{3}\sin \theta + 2\cos \theta = \frac{4}{r}$  is

$$\sqrt{3}\sin \left(\frac{\pi}{2} + \theta\right) + 2\cos \left(\frac{\pi}{2} + \theta\right) = \frac{k}{r} \dots(i)$$

Since, it is passing through  $(-1, \frac{\pi}{2})$

$$\therefore \sqrt{3}\sin \pi + 2\cos \pi = \frac{k}{-1} \Rightarrow k = 2$$

On putting  $k = 2$  in Eq. (i), we get

$$\sqrt{3}\cos \theta - 2\sin \theta = \frac{2}{r}$$

$$\Rightarrow 2 = \sqrt{3}r \cos \theta - 2r \sin \theta$$

346 (c)

Slope of refracted ray is

$$- \tan 60^\circ = -\sqrt{3}$$

It passes through (1, 0)

$$\therefore y = -\sqrt{3}(x - 1)$$

$$\Rightarrow \sqrt{3}x + y - \sqrt{3} = 0$$

347 (c)

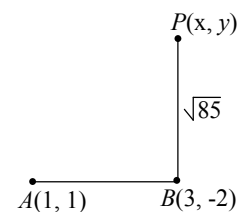
It is simple way to take a point from the option and finding the distance, which is equal to  $\sqrt{85}$

Taking point  $P(5, 7)$

$$BP = \sqrt{(5 - 3)^2 + (7 + 2)^2}$$

$$= \sqrt{4 + 81} = \sqrt{85}$$

Hence, option (c) is correct



348 (b)

Equation of the line  $\frac{ax}{c-1} + \frac{by}{c-1} + 1 = 0$  has two independent parameters. It can

pass through a fixed point if it contains only one independent parameter. Now, there must be one relation between  $\frac{a}{c-1}$  and  $\frac{b}{c-1}$  independent of  $a, b$  and  $c$  so that  $\frac{a}{c-1}$  can be expressed in terms of  $\frac{b}{c-1}$  and straight line contains only one independent parameter. Now, that given relation can be expressed as  $\frac{5a}{c-1} + \frac{4b}{c-1} = \frac{t-20c}{c-1}$  RHS in independent of  $c$  if  $t = 20$

349 (c)

On comparing given equation with standard equation, we get

$$a = 1, b = -1, c = -2, h = 0, g = -1/2, f = \lambda/2$$

Given equation represent a pair of straight line,

$$\therefore abc + 2 fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 2 + 0 - \frac{\lambda^2}{4} + \frac{1}{4} = 0$$

$$\Rightarrow \frac{\lambda^2}{4} = \frac{9}{4} \Rightarrow \lambda = \pm 3$$

350 (b)

The equation of given curve is

$$y = \sqrt{x} \dots(i)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\text{Slope of line at } (x_1, y_1), m_1 = \frac{1}{2\sqrt{x_1}}$$

and let line parallel to  $x$ -axis is  $y = k \dots(ii)$

Whose slope,  $m_2 = 0$

Since,  $45^\circ$  is the angle between the line and the curve.

$$\therefore \tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \Rightarrow 1 = \left| \frac{\frac{1}{2\sqrt{x_1}} - 0}{1} \right| \Rightarrow x_1 = \frac{1}{4}$$

$$\therefore y_1 = \frac{1}{2} \text{ [from Eq.(i)]}$$

$$\therefore \text{Required line is } y = \frac{1}{2} \text{ [from Eq.(ii)]}$$

351 (c)

(1) Let  $A$  and  $B$  be the points where the lines  $2x + 3y + 19 = 0$  meets the coordinates axes and let  $C$  and  $O$  be the points where the line  $9x + 6y - 17 = 0$  meet the coordinate axes

$$\text{Then, } OA = \frac{19}{2}, OB = \frac{19}{2},$$

$$OC = \frac{17}{9} \text{ and } OD = \frac{17}{6}$$

Thus, the segments  $AOC$  and  $BOD$  intersect at such that  $OA \cdot OC = OB \cdot OD$ . Hence,  $A, B, C, D$  are concyclic

(2) Distance of  $(2, -5)$  from the line  $3x + y + 5 = 0$  is

$$\frac{2 \times 3 - 5 + 5}{\sqrt{3^2 + 1^2}} = \frac{6}{\sqrt{10}}$$

and distance of  $(-1, 4)$  from the line  $3x + y + 5 = 0$  is

$$\frac{3(-1) + 4 + 5}{\sqrt{10}} = \frac{6}{\sqrt{10}}$$

Thus, the points are equidistant from the given line

Hence, both of these statements are correct

352 (a)

On comparing the given equation with the standard form of equation, we get  $a = 1$ ,  $h = 2$  and  $b = 1$

Let  $\theta$  is the angle between them, then

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\therefore \tan \theta = \frac{2\sqrt{2^2 - 1}}{1 + 1} = \frac{2\sqrt{4 - 1}}{2} = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

354 (d)

Here,  $a = 6$ ,  $2h = -1$ ,  $b = 4c$

$$\therefore m_1 + m_2 = \frac{1}{4c}, m_1 m_2 = \frac{6}{4c}$$

One line of given pair of line is  $3x + 4y = 0$

$$\text{Slope of line} = -\frac{3}{4} = m_1(\text{say})$$

$$\therefore -\frac{3}{4} + m_2 = \frac{1}{4c}$$

$$\Rightarrow m_2 = \frac{1}{4c} + \frac{3}{4}$$

$$\therefore \left(-\frac{3}{4}\right)\left(\frac{1}{4c} + \frac{3}{4}\right) = \frac{6}{4c}$$

$$\Rightarrow 1 + 3c = \frac{-6 \times 4}{3}$$

$$\Rightarrow 3c = -9 \Rightarrow c = -3$$

355 (b)

The equation  $4x^2 + 8xy + ky^2 - 9 = 0$  represents a pair of straight lines, if

$$(4)(k)(-9) - (-9)(4)^2 = 0 \Rightarrow k = 4$$

356 (b)

Slope of the line segment joining  $(-4, 6)$  and  $(8, 8)$  is

$$\frac{8 - 6}{8 - (-4)} = \frac{2}{12} = \frac{1}{6}$$

$\therefore$  Slope of line perpendicular to it.

$$m = -\frac{1}{1/6} = -6$$

As the line bisecting it.

$$\therefore \text{Mid point of this line is } \left(\frac{8-4}{2}, \frac{8+6}{2}\right) = (2, 7)$$

$\therefore$  Required equation is

$$y - 7 = -6(x - 2)$$

$$\Rightarrow y + 6x - 19 = 0$$

357 (c)

Let  $(h, k)$  be the point of intersection of the line  $x \cos \alpha + y \sin \alpha = a$  and  $x \sin \alpha - y \cos \alpha = b$ . Then,

$$h \cos \alpha + k \sin \alpha = a \quad \dots(i)$$

$$h \sin \alpha - k \cos \alpha = b \quad \dots(ii)$$

Squaring and adding (i) and (ii), we get

$$(h \cos \alpha + k \sin \alpha)^2 + (h \sin \alpha - k \cos \alpha)^2 = a^2 + b^2$$

$$\Rightarrow h^2 + k^2 = a^2 + b^2$$

Hence, locus of  $(h, k)$  is  $x^2 + y^2 = a^2 + b^2$

358 (a)

Equations of the bisectors of the angles between the lines  $x^2 - 2mxy - y^2 = 0$  are given by

$$\frac{x^2 - y^2}{1 - (-1)} = \frac{xy}{-m} \Rightarrow x^2 + \frac{2}{m}xy - y^2 = 0 \quad \dots(i)$$

Since (i) and  $x^2 - 2nxy - y^2 = 0$  represent the same pair of lines.

$$\therefore \frac{1}{1} = \frac{2/m}{-2n} = \frac{-1}{-1} \Rightarrow mn = -1 \Rightarrow mn + 1 = 0$$

359 (d)

Point of intersection of  $\frac{x}{a} + \frac{y}{b} = 1$  and

$$\frac{x}{b} + \frac{y}{a} = 1 \text{ is } \left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$$

$\therefore$  Equation of line joining  $(0, 0)$  and

$$\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right) \text{ is } x = y \text{ ie, } x - y = 0$$

360 (c)

Here,  $a = 4, b = 11$  and  $h = -12$

$$\therefore h^2 - ab = (-12)^2 - 4 \times 11 = 100$$

$\therefore$  The two lines represented by given equation will be real and distinct which represent a pair of straight lines passing through the origin.

### ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	A	A	C	C	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	D	D	B	B	C	A	D	C

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