

**Topic :-STRAIGHT LINES**

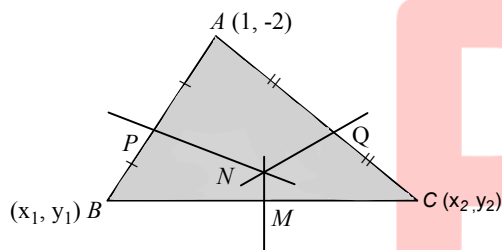
321 (a)

Let  $B(x_1, y_1)$  and  $C(x_2, y_2)$  be two vertices and

$P\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$  lies on perpendicular bisector  $x - y + 5 = 0$

$$\therefore \frac{x_1+1}{2} - \frac{y_1-2}{2} = -5$$

$$\Rightarrow x_1 - y_1 = -13 \dots(i)$$



Also,  $PN$  is perpendicular to  $AB$ .

$$\therefore \frac{y_1+2}{x_1-1} \times 1 = -1$$

$$\Rightarrow x_1 + y_1 = -1 \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x_1 = -7, y_1 = 6$$

$\therefore$  The coordinates of  $B$  are  $(-7, 6)$  Similarly, the coordinates of  $C$  are  $\left(\frac{11}{5}, \frac{2}{5}\right)$

Hence, the equation of  $BC$  is

$$y - 6 = \frac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$$

$$\Rightarrow y - 6 = \frac{-14}{23}(x + 7)$$

$$\Rightarrow 14x + 23y - 40 = 0$$

322 (b)

Points  $(a, 0)$  and  $(0, b)$  will satisfy the equation of line  $px - qy = r$

$$\Rightarrow ap = r, -bq = r$$

$$\therefore a + b = \frac{r}{p} - \frac{r}{q} = r \left( \frac{q-p}{pq} \right)$$

323 (d)

We have,

$$2x - y + 4 = 0 \text{ and } 6x - 3y - 5 = 0$$

$$\Rightarrow 2x - y + 4 = 0 \text{ and } 2x - y - 5/3 = 0$$

This distance between these two parallel lines is given by

$$d = \left| \frac{4 + 5/3}{\sqrt{2^2 + (-1)^2}} \right| = \frac{17\sqrt{5}}{15}$$

324 (b)

If the lines given by  $ax^2 + 5xy + 2y^2 = 0$  are mutually perpendicular, then

$$a + 2 = 0 \Rightarrow a = -2$$

326 (b)

Since, the coordinates of three vertices  $A, B$  and  $C$  are  $(\frac{5}{3}, -\frac{4}{3}), (0, 0)$  and  $(-\frac{2}{3}, \frac{7}{3})$  respectively, also the mid point of  $AC$  is  $(\frac{1}{2}, \frac{1}{2})$ , therefore the equation of line passing through  $(\frac{1}{2}, \frac{1}{2})$  and  $(0, 0)$  is given

by  $x - y = 0$ , which is the required equation of another diagonal, so

$$a = 1, b = -1, \text{ and } c = 0$$

327 (d)

Let  $m$  be the slope of required line

$$\therefore \left| \frac{m - (-1)}{1 + m(-1)} \right| = 1$$

$$\Rightarrow \frac{m + 1}{1 - m} = \pm 1$$

$$\Rightarrow m + 1 = 1 - m, m + 1 = -1 + m$$

$$\Rightarrow m = 0, m = \infty$$

$\therefore$  Equation of the line through  $(1, 1)$  is  $y - 1 = 0, x - 1 = 0$

328 (a)

Let the equation of line which is perpendicular to  $5x - 2y = 7$ , is

$$2x + 5y = \lambda \dots(i)$$

The point of intersection of given lines is  $(14, -9)$

Since, the Eq. (i) is passing through the point  $(14, -9)$

$$\therefore 2(14) + 5(-9) = \lambda \Rightarrow \lambda = -17$$

$\therefore$  Eq. (i) becomes

$$2x + 5y + 17 = 0$$

329 (a)

Let the vertices of the triangle be  $A(5, -2), B(-1, 2)$  and  $C(1, 4)$

The equation of the altitude through  $B(-1, 2)$  is

$$y + 2 = -(x - 5) \Rightarrow x + y - 3 = 0 \dots(i)$$

The equation of the altitude through  $C(1, 4)$

$$y - 2 = \frac{2}{3}(x + 1) \Rightarrow 2x - 3y + 8 = 0 \dots(ii)$$

Solving (i) and (ii), we obtain that the coordinates of the orthocentre are  $(1/5, 14/15)$

330 (a)

Since the origin and the point  $(1, -3)$  lie on the same side of  $x + 2y - 11 = 0$  and on the opposite side of  $3x - 6y - 5 = 0$ . Therefore, the bisector of the angle containing  $(1, -3)$  is the bisector of that angle which does not contain the origin and is given by

$$\frac{-x - 2y + 11}{\sqrt{5}} = - \left( \frac{-3x + 6y + 5}{\sqrt{45}} \right) \Rightarrow 3x = 19$$

**ALITER** Re-write the two equations in such a way that the values of the expressions on the left hand side of the equality for  $x = 1, y = -3$  become positive. Now, find the bisector corresponding to positive sign

331 (c)

For the two lines  $24x + 7y - 20 = 0$  and  $4x - 3y - 2 = 0$ , the angle bisectors are

$$\text{given by } \frac{24x + 7y - 20}{25} = \pm \frac{4x - 3y - 2}{5}$$

Talking positive sign, we get

$$2x + 11y - 5 = 0$$

$\therefore$  The given three lines are concurrent with one line bisecting the angle between the other two.

332 (b)

Let  $a$  and  $b$  be non-zero real numbers.

Therefore, the given equation

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0 \text{ implies either}$$

$$x^2 - 5xy + 6y^2 = 0$$

$$\Rightarrow (x - 2y)(x - 3y) = 0$$

$$\Rightarrow x = 2y \text{ and } x = 3y$$

Represent two straight lines passing through origin.

$$\text{or } ax^2 + by^2 + c = 0$$

When  $c = 0$  and  $a$  and  $b$  are of same signs, then

$$ax^2 + by^2 + c = 0$$

$$\Rightarrow x = 0 \text{ and } y = 0$$

Which is a point specified as the origin.

When  $a = b$  and  $c$  is of sign opposite to that of  $a$ , then

$$ax^2 + by^2 + c = 0 \text{ represents a circle.}$$

Hence, the given equation,

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$$

may represent two straight lines and a circle.

333 (c)

Equation of intersection of line is

$$(100x + 50y - 1) + \lambda(75x + 25y + 3) = 0$$

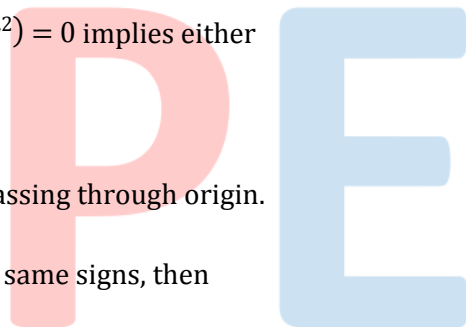
$$\Rightarrow (100 + 75\lambda)x + (50 + 25\lambda)y = -3\lambda \dots(i)$$

$$\Rightarrow \frac{x}{\frac{1 - 3\lambda}{100 + 75\lambda}} + \frac{y}{\frac{1 - 3\lambda}{50 + 25\lambda}} = 1$$

According to the given condition

$$\frac{1 - 3\lambda}{100 + 75\lambda} = \frac{1 - 3\lambda}{50 + 25\lambda}$$

$$\Rightarrow 50 = -50\lambda \Rightarrow \lambda = -1$$



∴ From Eq. (i), we get

$$25x + 25y - 4 = 0$$

334 (a)

The coordinates of the point dividing the line segment joining (2,3) and (-1,2) internally in the ratio 3 : 4 are

$$\left( \frac{3 \times -1 + 4 \times 2}{3 + 4}, \frac{3 \times 2 + 4 \times 3}{3 + 4} \right) = \left( \frac{5}{7}, \frac{18}{7} \right)$$

This point lies on the line  $x + 2y = \lambda$

$$\therefore \frac{5}{7} + \frac{36}{7} = \lambda \Rightarrow \lambda = \frac{41}{7}$$

335 (d)

Slopes of given lines are  $m_1 = \sqrt{3}$  and  $m_2 = \frac{1}{\sqrt{3}}$

$$\therefore \tan \theta = \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + 1} \right| = \left| \frac{3 - 1}{2\sqrt{3}} \right| = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ$$

336 (c)

The coordinates of the vertices of the rectangle are  $A(1,4), B(6,4), C(6,10), D(1,10)$ . The equation of diagonal  $AC$  is

$$y - 4 = \frac{10 - 4}{6 - 1}(x - 1) \Rightarrow 6x - 5y + 14 = 0$$

337 (d)

Let the equation of perpendicular line to the line

$$3x - 2y = 6 \text{ is } 3y + 2x = c \dots(i)$$

Since, it passes through (0, 2)

$$\therefore c = 6$$

On putting the value of  $c$  in Eq. (i) we get  $3y + 2x = 6$

$$\Rightarrow \frac{x}{3} + \frac{y}{2} = 1$$

Hence,  $x$ -intercept is 3.

338 (a)

$$\text{Given equation is } x^2 - 1005x + 2006 = 0$$

$$\Rightarrow (x - 2)(x - 1003) = 0$$

$$\Rightarrow x = 2, \quad x = 1003$$

∴ Required distance between the lines

$$= 1003 - 2 = 1001$$

339 (a)

We have,

$$\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$$

$$\Rightarrow (\sqrt{3}x - y)(x - \sqrt{3}y) = 0$$

$$\Rightarrow \sqrt{3}x - y = 0, x - \sqrt{3}y = 0$$

$$\Rightarrow y = \sqrt{3}x, y = \frac{1}{\sqrt{3}}x$$

These lines make  $60^\circ$  and  $30^\circ$  angles respectively with  $x$ -axis. If they are rotated about the origin by  $\pi/6$  i.e.  $30^\circ$  in anticlockwise direction, then they make  $90^\circ$  and  $60^\circ$  angles respectively with  $x$ -axis. So, their equations in new position are  $x = 0$  and  $y = \sqrt{3}x$ . The combined equation of these two lines is

$$x(\sqrt{3}x - y) = 0 \text{ or, } \sqrt{3}x^2 - xy = 0$$

340 **(b)**

Let the equations of the sides  $AB, BC, CD$  and  $DA$  of the parallelogram  $ABCD$  be respectively

$$3x - 4y + 1 = 0 \dots (i) \quad 4x - 3y - 2 = 0 \dots (ii)$$

$$3x - 4y + 3 = 0 \dots (iii) \quad 4x - 3y - 1 = 0 \dots (iv)$$

We know that the area of the parallelogram formed by the lines

$a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ ,  $a_1x + b_1y + d_1 = 0$  and  $a_2x + b_2y + d_2 = 0$  is given by

$$\frac{|(c_1d_1)(c_2 - d_2)|}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Hence, area  $\Delta$  of the given parallelogram is given by

$$\Delta = \frac{|(3 - 1) \times (-1 + 2)|}{\begin{vmatrix} 3 & -4 \\ 4 & -3 \end{vmatrix}} = \frac{2}{7} \text{ sq. units}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	B	B	B	D	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	D	C	D	A	A	B