

Topic :-STRAIGHT LINES

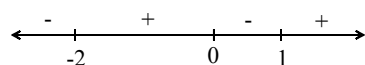
301 (b)

If the points $(\alpha, 2 + \alpha)$ and $(\frac{3\alpha}{2}, \alpha^2)$ are on the opposite sides of $2x + 3y - 6 = 0$, then

$$(2\alpha + 6 + 3\alpha - 6)(3\alpha + 3\alpha^2 - 6) < 0$$

$$\Rightarrow 15\alpha(\alpha^2 + \alpha - 2) < 0$$

$$\Rightarrow \alpha(\alpha + 2)(\alpha - 1) < 0 \Rightarrow \alpha \in (-\infty, -2) \cup (0, 1)$$



302 (c)

Let $y = m_1x$, $y = m_2x$ be the lines represented by $ax^2 + 2hxy + by^2 = 0$. Then,

$$m_1 + m_2 = \frac{-2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

Let $y = m_1'x$ and $y = m_2'x$ be new positions of $y = m_1x$ and $y = m_2x$ respectively. Then, $y = m_1x$ is perpendicular to $y = m_1'x$

$$\therefore m_1m_1' = -1 \Rightarrow m_1' = -\frac{1}{m_1}$$

Similarly, we have $m_2' = -\frac{1}{m_2}$

So, the new lines are $y = -\frac{1}{m_1}x$ and $y = -\frac{1}{m_2}x$ and their combined equation is

$$(m_1y + x)(m_2y + x) = 0$$

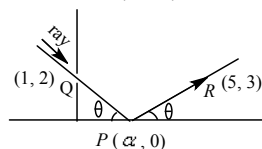
$$\Rightarrow m_1m_2y^2 + x^2 + xy(m_1 + m_2) = 0$$

$$\Rightarrow \frac{a}{b}y^2 + x^2 + xy\left(\frac{-2h}{b}\right) = 0$$

$$\Rightarrow bx^2 - 2hxy + ay^2 = 0$$

303 (c)

Here, in the figure it is shown that a ray of light passing through the point $Q(1, 2)$ and reflected from a point $P(\alpha, 0)$ on x -axis towards point $R(5, 3)$.



\therefore slope of incident ray (ie, before reflection) is given by

$$\tan(\pi - \theta) = \frac{0 - 2}{\alpha - 1}$$

$$\Rightarrow \tan \theta = \frac{2}{\alpha - 1} \dots(i)$$

Similarly, slope of reflected ray (*ie*, after reflection) is given by

$$\Rightarrow \tan \theta = \frac{3}{5 - \alpha} \dots(ii)$$

From Eq. (i) and (ii),

$$\frac{2}{\alpha - 1} = \frac{3}{5 - \alpha}$$

$$\Rightarrow 10 - 2\alpha = 3\alpha - 3 \Rightarrow \alpha = \frac{13}{5}$$

304 (c)

The equation of any line passing through $(1, -10)$ is $y + 10 = m(x - 1)$. Since makes equal angles, say θ , with the given lines. Therefore,

$$\tan \theta = \frac{m - 7}{1 + 7m} = -\frac{m - (-1)}{1 + m(-1)} \Rightarrow m = \frac{1}{3} \text{ or, } -3$$

Hence, the equations of third side are

$$y + 10 = \frac{1}{3}(x - 1) \text{ and } y + 10 = -3(x - 1)$$

i.e. $x - 3y - 31 = 0$ and $3x + y + 7 = 0$

ALITER Required lines are parallel to the angle bisectors

305 (c)

The line L is $x + y = 2$. The line perpendicular to L and passing through $(1/2, 0)$ is $x - y = 1$ and the equation of y -axis is $x = 0$. Solving these three equations in pairs we get the points as $(0, 2)$, $(0, -1/2)$ and $(5/4, 3/4)$. Therefore, the area Δ of the given triangle is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 0 & -\frac{1}{2} & 1 \\ \frac{5}{4} & \frac{3}{4} & 1 \end{vmatrix} = \frac{25}{16} \text{ sq. units}$$

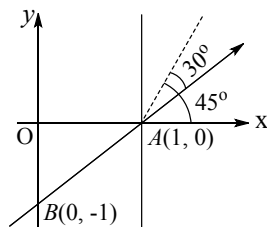
306 (c)

On comparing the given equation with standard equation, we get $a = 12$ and $b = a$, for perpendicular lines coefficient of $x^2 +$ coefficient of $y^2 = 0$

$$\therefore 12 + a = 0 \Rightarrow a = -12$$

307 (d)

From figure refracted ray makes an angle of 75° with positive direction of x -axis and passes through the point $(1, 0)$



∴ Its equation is
 $(y - 0) = \tan(45^\circ - 30^\circ)(x - 1)$
 or $y = (2 - \sqrt{3})(x - 1)$

308 (a)

The equation $12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$ will represent a pair of perpendicular lines

$$-72p - \frac{63}{2}q - 3q^2 + 81p - \frac{147}{2} = 0 \text{ and } 12 - p = 0$$

$$\Rightarrow 2q^2 + 21q - 23 = 0 \text{ and } p = 12$$

$$\Rightarrow q = 1 \text{ and } p = 12$$

309 (a)

Given, $|x + y| = 4$

If point (a, a) lies between the lines, then

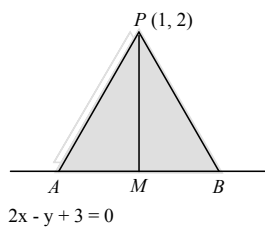
$$|a + a| = 4 \Rightarrow |a| = 2$$

310 (a)

Since, $AP = BP$ and PM is perpendicular to the line

$$2x - y + 3 = 0 \dots(i)$$

Where, M is the mid point AB



Therefore, its slope is $\left(-\frac{1}{2}\right)$

$$\therefore \text{Equation of line } PM \text{ is } y - 2 = -\frac{1}{2}(x - 1)$$

$$\Rightarrow 2y + x - 5 = 0 \dots(ii)$$

Solving Eqs. (i) and (ii), we get the mid point of AB is

$$M\left(-\frac{1}{5}, \frac{13}{5}\right)$$

311 (b)

Since, a, b, c are in HP

$$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$$

So, straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point $(1, -2)$

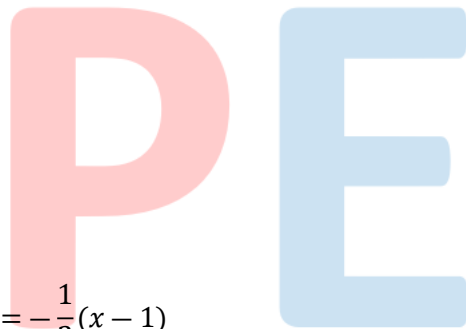
312 (c)

From the given equations, we get

$$m^2 + am + 2 = 0$$

Since, m is real, $a^2 \geq 8 \Rightarrow |a| \geq 2\sqrt{2}$

So, least value of $|a|$ is $2\sqrt{2}$



313 (c)

We have,

$$a^2x^2 + 2h(a + b)xy + b^2y^2 = 0 \dots(i)$$

$$ax^2 + 2hxy + by^2 = 0 \dots(ii)$$

The equation of the bisectors of the angles between the pair of lines given in (i) is

$$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a + b)} \Rightarrow \frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

This is same as the equation of the bisectors of the angles between the lines given in (ii). Thus, two pairs of straight lines are equally inclined to each other

316 (c)

We have,

$$xy + 2x + 2y + 4 = 0$$

$$\Rightarrow (x + 2)(y + 2) = 0 \Rightarrow x + 2 = 0, y + 2 = 0$$

Solving the equations of the sides of the triangle we obtain the coordinates of the vertices as $A(-2, 0), B(0, -2), C(-2, -2)$. Clearly, ΔABC is a right angled triangle with right angle at C .

Therefore, the centre of the circumcircle is the mid-point of AB whose coordinates are $(-1, -1)$

317 (d)

Focus is $|x| + |y| = 1$ which separately represents equation of straight lines.

318 (c)

Equation of line is

$$y = mx + 4$$

$$\therefore \text{Required distance} = \frac{4}{\sqrt{1 + m^2}}$$

320 (d)

$$\text{Let } x_1 = x, x_2 = xr, x_3 = xr^2$$

$$\text{and } y_1 = y, y_2 = yr, y_3 = yr^2$$

$\therefore x_1, x_2, x_3$ and y_1, y_2, y_3 are in GP.

$$\therefore \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_1 - y_3}{x_1 - x_3}$$

\therefore The points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) lies on a straight line.

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	C	C	C	D	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	C	B	A	C	D	C	C	D