DPP
DAILY PRACTICE PROBLEMS

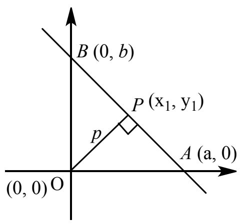
Class : XIth **Solutions** **Subject : MATHS**
Date : **DPP No. :10**

Topic :-STRAIGHT LINES

381 **(b)**

Equation of line is $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

Let P be the foot of perpendicular from the origin to the whose coordinate is (x_1, y_1).



Since, $OP \perp AB$

$\therefore$ Slope of $OP \times$ Slope of $AB = -1$

$\Rightarrow \left(\frac{y_1}{x_1}\right)\left(\frac{b}{-a}\right) = -1,$

$by_1 = ax_1$ ...(ii)

Since, P lies on the line AB, then

$\frac{x_1}{a}+\frac{y_1}{b}=1 \Rightarrow bx_1 + ay_1 = ab$ ...(iii)

From Eqs. (ii) and (iii), we get

$x_1 = \frac{ab^2}{a^2+b^2}$ and $y_1 = \frac{a^2b}{a^2+b^2}$

Now, $x_1^2 + y_1^2 = \left(\frac{ab^2}{a^2+b^2}\right)^2 + \left(\frac{a^2b}{a^2+b^2}\right)^2$

$\Rightarrow x_1^2 + y_1^2 = \frac{a^2b^2(a^2+b^2)}{(a^2+b^2)^2}$

$\Rightarrow x_1^2 + y_1^2 = \frac{a^2b^2}{(a^2+b^2)}$

$= \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$

But $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

$\therefore\ x_1^2+y_1^2=c^2$

Thus, the locus of $P(x_1,y_1)$ is

$x^2+y^2=c^2$

Which is the equation of circle.

382 **(a)**

Any line through A is given by

$(px+qy-1)+\lambda(qx+py-1)=0$

Which is passing through (p,q)

Hence, $\lambda=-\frac{(p^2+q^y-1)}{2pq-1}$

Thus, the required line is

$(px+qy-1)-\frac{(p^2+q^2-1)}{(2pq-1)}\cdot(qx+py-1)=0$

$\Rightarrow(2pq-1)(px+qy-1)-(p^2+q^2-1)(qx+py-1)=0$

383 **(a)**

Solving the given equations, we obtain that the vertices of the triangle formed by them are $A(0,4)$,$B(1,1)$ and $C(4,0)$

Now, $AB=\sqrt{10}=BC,CA=4\sqrt{2}$

Hence, triangle is isosceles

384 **(a)**

Image of (1,3) in the line $x+y-6=0$ is given by

$\frac{x-1}{1}=\frac{y-3}{1}=-2\left(\frac{1+3-6}{1^2+1^2}\right)\Rightarrow x=3,y=5$

Hence, the image of the given point has coordinates (3,5)

385 **(c)**

Given lines

$x\cos\alpha+\gamma\sin\alpha=p_1$ and, $x\cos\beta+\gamma\sin\beta=p_2$

Will be perpendicular, if the lines perpendicular to them are also perpendicular.

Clearly, perpendiculars drawn from the origin to the given lines make angles α and β respectively with x-axis. Therefore, angle between them is $|\alpha-\beta|$

Thus, the given lines will be perpendicular, if $|\alpha-\beta|=\frac{\pi}{2}$

387 **(c)**

Since, the given lines are concurrent.

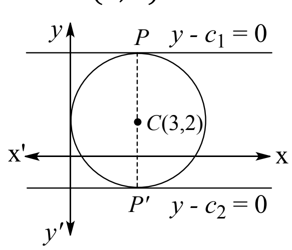
$\therefore\begin{vmatrix}a & k & 10\\ b & k+1 & 10\\ c & k+2 & 10\end{vmatrix}=0\Rightarrow 10\begin{vmatrix}a & k & 1\\ b & k+1 & 1\\ c & k+2 & 1\end{vmatrix}=0$

Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$\Rightarrow 10\begin{vmatrix}a & k & 1\\ b-a & 1 & 0\\ c-a & 2 & 0\end{vmatrix}=0$

$\Rightarrow 10[1(2b-2a-c+a)]=0$

$\Rightarrow 2b = a + c$
Hence, a,b and c are in AP
388 **(b)**
We have,
$$\text{Required distance} = 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = \frac{2}{\sqrt{10}}$$
389 **(c)**
Let $y = mx$ be a line represented by $ax^3 + bx^2y + cxy^2 + dy^3 = 0$. Then,
$$dm^3 + cm^2 + bm + a = 0 \quad \begin{bmatrix}\text{Putting } y = mx \text{ in } ax^3 + bx^2y \\ +cxy^2 + dy^3 = 0\end{bmatrix}$$
Let m_1,m_2,m_3 be the roots of this equation. Then,
$$m_1 + m_2 + m_3 = -\frac{c}{d}$$
$$m_1m_2 + m_2m_3 + m_3m_1 = \frac{b}{d}$$
$$m_1m_2m_3 = -\frac{a}{d}$$
Thus, there are three lines viz. $y = m_1x$,$y = m_2x$,$y = m_3x$ represented by the given equation.
Suppose $y = m_1x$ and $y = m_2x$ make complementary angles with x-axis. Then,
$m_1m_2 = 1$
Putting $m_1m_2 = 1$ in $m_1m_2m_3 = -\frac{a}{d}$, we get
$$m_3 = -\frac{a}{d}$$
Since m_3 is a root of the equation $dm^3 + cm^2 + bm + a = 0$
$$\therefore d\left(-\frac{a}{d}\right)^3 + c\left(-\frac{a}{d}\right)^2 + b\left(-\frac{a}{d}\right) + a = 0$$
$\Rightarrow -a^3d + a^2cd - abd^2 + ad^3 = 0$
$\Rightarrow -a^2 + ac - bd + d^2 = 0$
$\Rightarrow a(c - a) = d(b - d) \Rightarrow a(a - c) = d(d - b)$
390 **(a)**
Let the lines are $y = m_1x + c_1$ and $y = m_2x + c_2$
Since, pair of straight lines are parallel to x-axis
$\therefore m_1 = m_2 = 0$
and the lines will be $y = c_1$ and $y = c_2$
Given circle is $x^2 + y^2 - 6x - 4y - 12 = 0$
Centre (3, 2) and radius $= 5$



Here, the perpendicular drawn from centre to the lines are CP and CP''

$\therefore CP = \frac{2 - c_1}{\sqrt{1}} = \pm 5$

$\Rightarrow 2 - c_1 = \pm 5$

$\Rightarrow \ c_1 = 7$ and $c_1 = -3$

Hence, the lines are

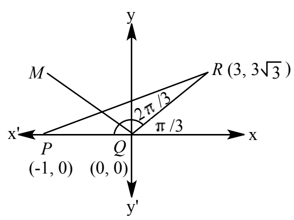
$y - 7 = 0, y + 3 = 0, ie, (y - 7)(y + 3) = 0$

$\therefore$ Pair of straight lines is $y^2 - 4y - 21 = 0$

391 **(a)**

Now, slope of $QR = \frac{3\sqrt{3} - 0}{3 - 0} = \sqrt{3} = \tan\theta$

$\Rightarrow \ \theta = \frac{\pi}{3}$



$\therefore$ The angle between PQR is $\frac{2\pi}{3}$, so the line QM makes an angle $\frac{2\pi}{3}$ from positive direction of x-axis.

Slope of the line $QM = \tan\frac{2\pi}{3} = -\sqrt{3}$

Hence, equation of line QM is $y = -\sqrt{3}x$

or $\sqrt{3}x + y = 0$

392 **(a)**

Let $ay^4 + bxy^3 + cx^2y^2 + dx^3y + ex^4 = (ax^2 + pxy - ay^2)(x^2 + qxy + y^2)$

On comparing the coefficient of similar terms, we get

$b = aq - p, c = -pq, d = aq + p, e = -a$

Now, $b + d = 2aq, \ \ e - a = -2a$

$ad + be = 2ap, a + c + e = -pq$

$\therefore \ (b + d)(ad + be) = -(e - a)^2(a + c + e)$

$\Rightarrow (b + d)(ad + eb) + (e - a)^2(a + c + e) = 0$

393 **(d)**

Given equation is

$3x^2 + xy - y^2 - 3x + 6y + k = 0$

Here, $a = 3, b = -1, h = \frac{1}{2}, g = -\frac{3}{2} \ f = 3, c = k,$

Given equation represents a pair of straight line, if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore 3(-1)(k) + 2 \times 3 \times \left(-\frac{3}{2}\right) \times \frac{1}{2} - 3(3)^2 + 1\left(\frac{-3}{2}\right)^2 - k\left(\frac{1}{2}\right)^2 = 0$

$\Rightarrow \ -3k - \frac{9}{2} - 27 + \frac{9}{4} - \frac{k}{4} = 0 \Rightarrow k = -9$

394 **(b)**

Let (h,k) be the coordinates of the vertex. Then, the height of the triangle is the length of the perpendicular from (h,k) on $x = a$ i.e. $|h - a|$

Since the area of the triangle is a^2

$$\therefore \frac{1}{2}(2\,a)|h - a| = a^2$$

$$\Rightarrow |h - a| = a$$

$$\Rightarrow h - a = \pm\, a \Rightarrow h = 0, h = 2\,a$$

Hence, the vertex lies on $x = 0$ or, $x = 2a$

395 **(a)**

The distance of the point $(-2, 3)$ from the line $x - y = 5$ is

$$p = \left|\frac{-2 - 3 - 5}{\sqrt{(1)^2 + (-1)^2}}\right|$$

$$= \left|\frac{-10}{\sqrt{2}}\right| = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

396 **(b)**

Here, $h = -\frac{1}{2}, a = 1, b = -6$

$$\therefore \tan = \left|2\frac{\sqrt{\frac{1}{4} + 6}}{1 - 6}\right| = \frac{2\sqrt{\frac{25}{4}}}{-5} = |-1|$$

$$\therefore \theta = \tan^{-1}(1) = 45°$$

397 **(c)**

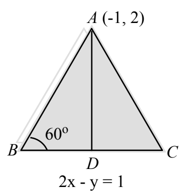
On comparing the given equation with standard equation, we get $a = 12$ and $b = a$. We also know, if pair of straight lines is perpendicular, then coefficient of $x^2 +$ coefficient of $y^2 = 0$ or $a + b = 0$

$$\therefore 12 + a = 0 \Rightarrow a = -12$$

398 **(a)**

$$\because AD = \left|\frac{-2 - 2 - 1}{\sqrt{(2)^2 + (-1)^2}}\right| = \left|\frac{-5}{\sqrt{5}}\right| = \sqrt{5}$$

and in $\Delta\, ABD$ $\tan 60° = \frac{AD}{BD}$



$$\Rightarrow \sqrt{3} = \frac{\sqrt{5}}{BD} \Rightarrow BD = \sqrt{\frac{5}{3}}$$

$$\therefore BC = 2BD = 2\sqrt{\frac{5}{3}} = \sqrt{\frac{20}{3}}$$

399 **(b)**

The equation of any line parallel to $2x + 6y + 7 = 0$ is $2x + 6y + k = 0$. This meets the axes at $A(-k/2, 0)$ and $B(0, -k/6)$

Now,

$AB = 10$

$\Rightarrow \sqrt{\frac{k^2}{4} + \frac{k^2}{36}} = 10$

$\Rightarrow \sqrt{\frac{10\,k^2}{36}} = 10$

$\Rightarrow 10\,k^2 = 3600 \Rightarrow k = \pm\, 6\sqrt{10}$

Hence, there are two lines given by $2x + 6y \pm 6\sqrt{10} = 0$s

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	A	A	C	A	C	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	D	B	A	B	C	A	B	A