

Topic :-STRAIGHT LINES

201 (a)

The equation of the family of lines is

$$(\lambda + \mu)x + (2\lambda + \mu)y = \lambda + 2\mu$$

$$\Rightarrow \lambda(x + 2y - 1) + \mu(x + y - 2) = 0$$

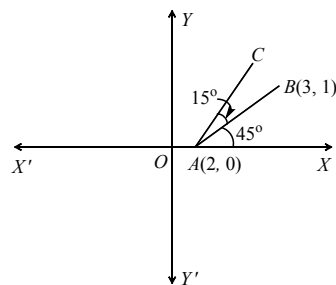
Clearly, it represents a family of lines passing through the intersection of the lines $x + 2y - 1 = 0$ and $x + y - 2 = 0$ i.e. $(3, -1)$

202 (a)

We have,

$$\text{Slope of } AB = \frac{1-0}{3-2} = 1 \Rightarrow \angle BAX = \frac{\pi}{4}$$

But, $\angle BAC = 15^\circ$. Therefore, $\angle CAX = 60^\circ$



So, the equation of AC is

$$y - 0 = \tan 60^\circ(x - 2)$$

$$\Rightarrow y = \sqrt{3}x - 2\sqrt{3} \Rightarrow \sqrt{3}x - y = 2\sqrt{3}$$

203 (c)

The sides of the triangle are $y = 1$ and the pair of lines $x^2 + 7xy + 2y^2 = 0$

Clearly, one vertex is $(0, 0)$ and the y -coordinates of each of the other two vertices is 1.

On putting $y = 1$ in the second equation, we get

$$x^2 + 7x + 2 = 0$$

If x_1 and x_2 are the roots of this equation, then

$$x_1 + x_2 = -7$$

$$\therefore \text{Centroid, } G = \left(\frac{0 + x_1 + x_2}{3}, \frac{0 + 1 + 1}{3} \right)$$

$$= \left(-\frac{7}{3}, \frac{2}{3} \right)$$

204 (c)

Let equation of line parallel to $3x - y = 7$ be $3x - y = \lambda$.

The passes through (1, 2)

$$\therefore 3 - 2 = \lambda \Rightarrow \lambda = 1$$

$$\therefore \text{Line is } 3x - y = 1$$

The point of intersection of $x + y + 5 = 0$ and $3x - y = 1$ is $(-1, -4)$

$$\therefore \text{Distance between } (1, 2) \text{ and } (-1, -4)$$

$$= \sqrt{(2)^2 + (6)^2} = \sqrt{40}$$

206 (a)

Here, $a = 1, b = 4, g = \frac{3}{2}, f = 3, h = 2$ and $c = -4$

$$\text{Then, required distance} = 2 \sqrt{\frac{\frac{9}{4} + 4}{5}}$$

$$= \frac{2\sqrt{25}}{2\sqrt{5}} = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

207 (d)

Equation of pair straight lines is $xy - x - y + 1 = 0$

$$\Rightarrow (x - 1)(y - 1) = 0$$

$$\Rightarrow x - 1 = 0 \text{ or } y - 1 = 0$$

The intersection points of $x - 1, y - 1 = 0$ is $(1, 1)$

$$\therefore \text{Lines } x - 1 = 0, y - 1 = 0 \text{ and } ax + 2y - 3 = 0 \text{ are concurrent}$$

$$\therefore \text{The intersecting points of first two lines lies on the third line } ax + 2y - 3 = 0$$

$$\therefore a + 2 - 3 = 0 \Rightarrow a = 1$$

208 (a)

Any point on $x + y = 4$ is $(t, 4 - t)$. It is at a unit distance from the line $4x + 3y - 10 = 0$

$$\therefore \left| \frac{4t + 3(4 - t) - 10}{\sqrt{4^2 + 3^2}} \right| = 1 \Rightarrow t = 3, -7$$

Hence, the required points are $(3, 1)$ and $(-7, 11)$

209 (c)

The equation of bisector of acute angle formed between the lines $4x - 3y + 7 = 0$ at

$$3x - 4y + 14 = 0, \text{ is}$$

$$4x - 3y + 7$$

$$\frac{\sqrt{16 + 9}}{3x - 4y + 14} = -\frac{\sqrt{16 + 9}}{4x - 3y + 7}$$

$$\Rightarrow 7x - 7y + 21 = 0$$

$$\Rightarrow x - y + 3 = 0$$

210 (d)

The equations will represent the same line if

$$\frac{b^3 - c^3}{b - c} = \frac{c^3 - a^3}{c - a} = \frac{a^3 - b^3}{a - b}$$

$$\Rightarrow b^2 + bc + c^2 = c^2 + ca + a^2 = a^2 + ab + b^2$$

$$\Rightarrow b^2 + bc + c^2 = c^2 + ca + a^2 \text{ and } b^2 + bc + c^2 = a^2 + ab + b^2$$

$$\Rightarrow b^2 - a^2 + bc - ca = 0 \text{ and } c^2 - a^2 + bc - ab = 0$$

$$\Rightarrow (b-a)(b+a+c) = 0 \text{ and } (c-a)(c+a+b) = 0$$

$$\Rightarrow a+b+c = 0$$

211 (a)

Given lines are $x + y = 4$ and $2x + 2y = 5$ or $x + y = \frac{5}{2}$

The distance between two parallel lines,

$$d = \frac{4 - \frac{5}{2}}{\sqrt{1^2 + 1^2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4} > 1$$

Hence, no point lies in it.

213 (a)

Given lines are concurrent, if

$$\begin{vmatrix} 2 & 1 & -1 \\ a & 3 & -3 \\ 3 & 2 & -2 \end{vmatrix} = 0$$

This is true for all values of a , because C_2 and C_3 are identical

214 (b)

Let (h, k) be the centroid of the triangle having vertices $A(\cos \alpha, -\cos \alpha)$ and $C(1, 2)$. Then,

$$h = \frac{\cos \alpha + \sin \alpha + 1}{3} \text{ and } k = \frac{\sin \alpha - \cos \alpha + 2}{3}$$

$$\Rightarrow 3h - 1 = \cos \alpha + \sin \alpha \text{ and } 3k - 2 = \sin \alpha - \cos \alpha$$

$$\Rightarrow (3h - 1)^2 + (3k - 2)^2 = 2 \quad [\text{Squaring and adding}]$$

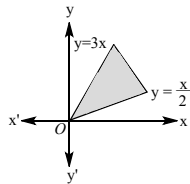
$$\Rightarrow 9(h^2 + k^2) - 6h - 12k + 3 = 0$$

$$\Rightarrow 3(h^2 + k^2) - 2h - 4k + 1 = 0$$

Hence, the locus of (h, k) is $3(x^2 + y^2) - 2x - 4y + 1 = 0$

215 (b)

The graph of equations $x - 2y = 0$ and $3x - y = 0$ is as shown in the figure. Since, given point (a, a^2) lies in the shaded region.



$$\text{Then, } a^2 - \frac{a}{2} > 0$$

$$\text{and } a^2 - 3a < 0$$

$$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$$

$$\text{and } a \in (0, 3)$$

$$\Rightarrow a \in \left(\frac{1}{2}, 3\right)$$

216 (d)

The two pairs of lines are

$$ax^2 + 2hxy - ay^2 = 0 \dots(i)$$

$$hx^2 - 2axy - hy^2 = 0 \dots(ii)$$

Clearly, these two equations represent two pairs of lines such that the lines in each pair are mutually perpendicular.

The combined equation of the bisectors of the angles between the lines given in (i) is

$$\frac{x^2 - y^2}{a + a} = \frac{xy}{h} \Rightarrow hx^2 - 2axy - hy^2 = 0$$

Clearly it is same as (ii).

Thus, each pair bisects the angle between the other pair.

Also, lines of one pair are equally inclined to the lines of the other pair

217 **(a)**

\therefore Line $ax + by + c = 0$ passes through $(1, -2)$

$$\therefore a - 2b + c = 0$$

$$\Rightarrow 2b = a + c$$

$\Rightarrow a, b, c$ are in AP.

218 **(d)**

The diagonal through B passes through the mid-point of AC . The coordinates of the mid point of AC are

$$\left(\frac{\sqrt{3} + 1}{2}, \frac{\sqrt{3} + 3}{2} \right)$$

\therefore equation of the diagonal through B is

$$y - 2 = \frac{\left(\frac{\sqrt{3} + 3}{2} \right) - 2}{\left(\frac{\sqrt{3} + 1}{2} \right) - (\sqrt{3} + 1)} (x - \sqrt{3} - 1)$$

$$\Rightarrow y = x(\sqrt{3} - 2) + (1 + \sqrt{3})$$

219 **(c)**

Since, the given three lines are concurrent, then

$$\begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ k & 5 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 4(3 - 25) - 3(-3 - 5k) - 1(5 + k) = 0$$

$$\Rightarrow -88 + 9 + 15k - 5 - k = 0$$

$$\Rightarrow 14k = 84 \Rightarrow k = 6$$

220 **(b)**

On comparing the given equation with

$$ax^2 + 2hxy + by^2 = 0, \text{ we get}$$

$$a = 1, 2h = 2h \text{ and } b = 2$$

Let the slope of the lines are m_1 and m_2 .

$$\therefore m_1 : m_2 = 1 : 2$$

Let $m_1 = m$ and $m_2 = 2m$

$$\therefore m_1 + m_2 = -\frac{2h}{a} \Rightarrow m + 2m = -h \Rightarrow h = -3m \dots(i)$$

$$\text{and } m_1 m_2 = \frac{a}{b} \Rightarrow m \cdot 2m = \frac{1}{2} \Rightarrow m = \pm \frac{1}{2} \dots(ii)$$

From Eqs. (i) and (ii), we get

$$h = \pm \frac{3}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	A	A	D	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	B	B	D	A	D	C	B

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