

Topic :-STATISTICS

101 (c)

We have,

$$n_1 = 7, \bar{X}_1 = 10, n_2 = 3, \bar{X}_2 = 5$$

$$\therefore \text{Combined mean} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2} = \frac{85}{10} = 8.5$$

103 (b)

From the given table, it is clear that required mode=6

104 (a)

Let x_1, x_2, \dots, x_n be n observations

$$\begin{aligned} \therefore M &= \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \\ &= \frac{x_1 + x_2 + \dots + x_{n-4} + x_{n-3} + x_{n-2} + x_{n-1} + x_n}{n} \\ \Rightarrow nM &= a + x_{n-3} + x_{n-2} + x_{n-1} + x_n \\ \Rightarrow \frac{nM - a}{4} &= \frac{x_{n-3} + x_{n-2} + x_{n-1} + x_n}{4} \end{aligned}$$

105 (b)

The mean of the series $a, a + d, a + 2d, \dots, a + 2nd$ is

$$\begin{aligned} \bar{X} &= \frac{1}{2n+1} [a + a + d + a + 2d + \dots + a + 2nd] \\ \Rightarrow \bar{X} &= \frac{1}{2n+1} \left\{ \frac{2n+1}{2} (a + a + 2nd) \right\} = a + nd \end{aligned}$$

\therefore Mean deviation from mean

$$\begin{aligned} \text{M.D.} &= \frac{1}{2n+1} \sum_{r=0}^{2n} |(a + rd) - (a + nd)| \\ \Rightarrow \text{M.D.} &= \frac{1}{2n+1} \sum_{r=0}^{2n} |r - n|d \\ \Rightarrow \text{M.D.} &= \frac{1}{2n+1} \{2d(1 + 2 + \dots + n)\} = \frac{n(n+1)}{2n+1} d \end{aligned}$$

106 (c)

Let x_1, x_2, \dots, x_n be n values of variable X . Then,

$$\bar{X} = \frac{1}{n} \sum x_i$$

Let $y_1 = x_1 + 1, y_2 = x_2 + 2, y_3 = x_3 + 3, \dots, y_n = x_n + n$. Then, the mean of the new series is given by

$$\bar{X}' = \frac{1}{n} \sum y_i$$

$$\Rightarrow \bar{X}' = \frac{1}{n} \sum_i (x_i + i)$$

$$\Rightarrow \bar{X}' = \frac{1}{n} \sum_i x_i + \frac{1}{n} (1 + 2 + 3 + \dots + n)$$

$$\Rightarrow \bar{X}' = \bar{X} + \frac{1}{n} \cdot \frac{n(n+1)}{2} = \bar{X} + \frac{n+1}{2}$$

107 (c)

$$\text{Mean} = \frac{\sum_{i=1}^n (x_i + 2i)}{n} = \frac{\sum_{i=1}^n x_i + 2(1 + 2 + \dots + n)}{n}$$

$$\bar{x} + \frac{2n(n+1)}{2n} = \bar{x} + (n+1)$$

109 (b)

If mean, median and mode coincides, then their is a symmetrical distribution

111 (d)

Let $x_i/f_i; i = 1, 2, \dots, n$ be a frequency distribution.

Then,

$$\text{S.D.} = \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{X})^2} \text{ and M.D.} = \frac{1}{N} \sum_{i=1}^n f_i |x_i - \bar{X}|$$

Let $|x_i - \bar{X}| = z_i; i = 1, 2, \dots, n$. Then,

$$(\text{S.D.})^2 - (\text{M.D.})^2 = \frac{1}{N} \sum_{i=1}^n f_i z_i^2 - \left(\frac{1}{N} \sum_{i=1}^n f_i z_i \right)^2 = \sigma_z^2 \geq 0$$

$\Rightarrow \text{S.D.} \geq \text{M.D.}$

112 (d)

We have,

$$Q_3 = 17 \text{ and } Q_1 = 10 \Rightarrow \text{Q.D.} = \frac{1}{2}(Q_3 - Q_1) = 3.5$$

113 (c)

When the origin is changed, then the coefficient of correlation is unsalted.

114 (c)

$$\bar{x} = \frac{31 + 32 + 33 + \dots + 47}{47} = \frac{663}{47} = 39$$

$$\begin{aligned} \therefore \sum_{i=1}^{17} (x_i - \bar{x})^2 &= (31 - 39)^2 + (32 - 39)^2 \\ &+ (33 - 39)^2 + (34 - 39)^2 + (35 - 39)^2 \\ &+ (36 - 39)^2 + (37 - 39)^2 + (38 - 39)^2 \\ &+ (39 - 39)^2 + (40 - 39)^2 + (41 - 39)^2 \end{aligned}$$

$$\begin{aligned}
&+ (42 - 39)^2 + (43 - 39)^2 + (44 - 39)^2 \\
&+ (45 - 39)^2 + (46 - 39)^2 + (47 - 39)^2 \\
&= 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 + 0 + 1 \\
&+ 4 + 9 + 16 + 25 + 36 + 49 + 64 \\
&= 408
\end{aligned}$$

Hence, standard deviation = $\sqrt{\frac{408}{17}} = \sqrt{24} = 2\sqrt{6}$

117 (b)

Arranging the terms in increasing order

Value x	Frequency f	Commulative frequency
7	2	2
8	1	3
9	5	8
10	4	12
11	6	18
12	1	19
13	3	22

$\therefore N = 22$

\therefore Median number = $\frac{N+1}{2} = 11.5$

Which comes under the cumulative frequency the corresponding value of x will be the median i.e.,

Median = 10

118 (d)

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1} = 2^{2n+1}$$

Now, ${}^{2n+1}C_0 = {}^{2n+1}C_{2n+1}$,

$${}^{2n+1}C_1 = {}^{2n+1}C_{2n} \dots {}^{2n+1}C_r = {}^{2n+1}C_{2n-r+1}$$

So, sum of first $(n+1)$ terms = sum of last $(n+1)$ terms

$$\Rightarrow {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n = 2^{2n}$$

$$\Rightarrow \frac{{}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n}{n+1} = \frac{2^{2n}}{(n+1)}$$

119 (c)

If both two regression lines are perpendicular, then correlation coefficient will be zero.

120 (b)

The mean of the series $a, a + d, \dots, a + 2nd$ is

$$\bar{x} = \frac{1}{2n+1} [a + a + d + a + 2d + \dots + a + 2nd]$$

$$= \frac{1}{2n+1} \left[\frac{2n+1}{2} (a + a + 2nd) \right] = a + nd$$

\therefore Mean deviation from mean

$$\begin{aligned}
&= \frac{1}{2n+1} \sum_{r=0}^{2n} |(a+rd) - (a+nd)| \\
&= \frac{1}{2n+1} \sum_{r=0}^{2n} (r-n)d \\
&= \frac{1}{2n+1} 2d(1+2+\dots+n) \\
&= \frac{n(n+1)}{2n+1} d
\end{aligned}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	B	A	B	C	C	D	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	C	C	A	C	B	D	C	B