

## Topic :-STATISTICS

22 (a)

$$\bar{x} = \frac{1 + 2 + 3 + \dots + n}{n} = \frac{(n + 1)}{2}$$

Variance,  $\sigma^2 = \frac{\sum (x_i)^2}{n} - (\bar{x})^2$

$$= \frac{\sum n^2}{n} - \left(\frac{n + 1}{2}\right)^2$$
$$= \frac{n(n + 1)(2n + 1)}{6n} - \left(\frac{n + 1}{2}\right)^2$$
$$= \frac{n^2 - 1}{12}$$

25 (a)

Coefficient of skewness =  $\frac{Q_3 - Q_1 - 2(\text{median})}{Q_3 - Q_1}$

$$= \frac{25.2 + 14.6 - 2(18.8)}{25.2 - 14.6}$$
$$= \frac{2.2}{10.6} = 0.20$$

27 (a)

Let  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_n$  be two series of observations with geometric means  $G_1$  and  $G_2$  respectively

Then,

$$G_1 = (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{1/n} \text{ and } G_2 = (y_1 \cdot y_2 \cdot \dots \cdot y_n)^{1/n}$$

Since  $G$  is the geometric mean of the ratios of the corresponding observations

$$\therefore G = \left(\frac{x_1}{y_1} \cdot \frac{x_2}{y_2} \cdot \dots \cdot \frac{x_n}{y_n}\right)^{1/n} = \frac{(x_1 x_2 \dots x_n)^{1/n}}{(y_1 \cdot y_2 \dots y_n)^{1/n}} = \frac{G_1}{G_2}$$

28 (d)

We know that,

$$r = \pm \sqrt{b_{yx} \cdot b_{xy}}$$

Also we know that sign of  $r, b_{xy}, b_{yx}$  are all same.

$$\therefore r = (\text{sign of } b_{yx}) \sqrt{b_{yx} \cdot b_{xy}}$$

30 (b)

Let  $Y = \frac{aX+b}{c}$ . Then,  $\bar{Y} = \frac{1}{c}(a\bar{X} + b)$

$$\therefore Y - \bar{Y} = \frac{a}{c}(X - \bar{X})$$

$$\Rightarrow \frac{1}{N} \sum (Y - \bar{Y})^2 = \frac{a^2}{c^2} \frac{1}{N} \sum (X - \bar{X})^2$$

$$\therefore \sigma_Y = \sqrt{\frac{a^2}{c^2} \times \frac{1}{N} \sum (X - \bar{X})^2} = \sqrt{\frac{a^2}{c^2} \sigma^2} = \left| \frac{a}{c} \right| \sigma$$

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**(d)**

We have,

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_{n-1} + x_n}{n}$$

$$\Rightarrow n\bar{X} = x_1 + x_2 + \dots + x_{n-1} + x_n$$

Let  $\bar{Y}$  be the new mean when  $x_2$  is replaced by  $\lambda$ . Then,

$$\bar{Y} = \frac{x_1 + \lambda + x_3 + \dots + x_{n-1} + x_n}{n}$$

$$\Rightarrow \bar{Y} = \frac{(x_1 + x_2 + \dots + x_n) - x_2 + \lambda}{n}$$

$$\Rightarrow \bar{Y} = \frac{n\bar{X} - x_2 + \lambda}{n}$$

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**(b)**

Let  $x_1, x_2, \dots, x_n$  be  $n$  values of  $x$ .

Then,  $\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$  ... (i)

The variable  $ax + b$  takes values  $ax_1 + b, ax_2 + b, \dots, ax_n + b$  with mean  $a\bar{x} + b$ .

$$\therefore SD \text{ of } (ax + b) = \sqrt{\frac{1}{n} \sum_{i=1}^n \{(ax_i + b) - (a\bar{x} + b)\}^2}$$

$$= \sqrt{a^2 \cdot \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$= |a| \sigma$$

**Alternate**  $SD(ax + b) = SD(ax) + SD(b)$

$$= |a|SD(x) + 0$$

$$= |a| \sigma$$

33

**(d)**

$$\text{Mean} = \frac{1}{10}[(x_1 + x_2 + \dots + x_{10}) + (4 + 8 + \dots + 40)]$$

$$= \frac{1}{10}(x_1 + x_2 + \dots + x_{10}) + \frac{4}{10}(1 + 2 + \dots + 10)$$

$$= 20 + \frac{4 \times 10 \times 11}{10 \times 2} = 42$$

35

**(d)**

$$\begin{aligned}\bar{x} &= \frac{1}{2n+1}[a + (a+d) + \dots + (a+2nd)] \\ &= \frac{1}{2n+1}[(2n+1)a + d(1+2+\dots+2n)] \\ &= a + d \frac{2n}{2} \cdot \frac{(2n+1)}{2n+1} = a + nd \\ \therefore \text{MD from mean} &= \frac{1}{2n+1} \sum |x_1 - \bar{x}| \\ &= \frac{1}{2n+1} 2|d|(1+2+\dots+n) \\ &= \frac{n(n+1)|d|}{(2n+1)}\end{aligned}$$

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**(b)**

$$\text{Given, } \sigma_{10}^2 = \frac{99}{12} = \frac{33}{4}$$

$$\Rightarrow \sigma_{10} = \frac{\sqrt{33}}{2}$$

$$\text{SD of required series} = 3\sigma_{10} = \frac{3\sqrt{33}}{2}$$

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**(b)**

We know that,

$$\text{var}(aX + b) = a^2 \text{var}(X)$$

$$\therefore \text{var}\left(\frac{aX + b}{c}\right) = \left(\frac{a}{c}\right)^2 \text{var}(X) = \frac{a^2}{c^2} \sigma^2$$

$$\therefore \text{SD} = \sqrt{\text{var}\left(\frac{aX + b}{c}\right)} = \left|\frac{a}{c}\right| \sigma$$

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**(b)**

We have,

$$\sigma^2 = \frac{1}{2n+1} \sum_{r=0}^{2n} \{(a+rd) - (a+nd)\}^2$$

$$\Rightarrow \sigma^2 = \frac{2d^2}{2n+1} \{1^2 + 2^2 + \dots + n^2\}$$

$$\Rightarrow \sigma^2 = \frac{n(n+1)}{3} d^2 \Rightarrow \sigma = \sqrt{\frac{n(n+1)}{3}} d$$

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**(a)**

Given that, mean=5, median=6

For a moderately skewed distribution, we have

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean}$$

$$\Rightarrow \text{Mode} = 3(6) - 2(5) = 8$$

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	D	C	B	A	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	D	B	D	B	B	B	B	A

PE