DPP

DAILY PRACTICE PRODUCTS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :9

Topic :-SEQUENCES AND SERIES

1 **(c)**

Put $X = \frac{x-y}{x}$ in $\log_e(1-X) = -\left[\frac{X}{1} + \frac{X^2}{2} + \frac{X^3}{3} + \ldots\right]$

We get

$\log_e\left(1 - \frac{x-y}{x}\right) = -\left[\frac{x-y}{x} + \frac{1}{2}\left(\frac{x-y}{x}\right)^2 + \frac{1}{3}\left(\frac{x-y}{x}\right)^3 + \ldots\right]$

$\Rightarrow \frac{x-y}{x} + \frac{1}{2}\left(\frac{x-y}{x}\right)^2 + \frac{1}{3}\left(\frac{x-y}{x}\right)^3 + \ldots = -\log_e\left(\frac{y}{x}\right) = \log_e\frac{x}{y}$

2 **(b)**

Let $x = n$, $y = n+1$ and $z = n+2$, where n is a positive integer.

$\therefore \log_e\sqrt{x} + \log_e\sqrt{z} + \left(\frac{1}{2xz+1}\right) + \frac{1}{3}\left(\frac{1}{2xz+1}\right)^3 + \frac{1}{5}\left(\frac{1}{2xz+1}\right)^5 + \ldots$

$= \log_e\sqrt{xz} + \frac{1}{2}\log_e\left[\frac{1+\frac{1}{2xz+1}}{1-\frac{1}{2xz+1}}\right]$

$= \log_e\sqrt{xz} + \frac{1}{2}\log_e\left(\frac{2xz+2}{2xz}\right)$

$= \log_e\sqrt{n(n+2)} + \log_e\sqrt{\frac{n(n+2)+1}{n(n+2)}}$

$= \log_e\sqrt{(n+1)^2} = \log_e(n+1)$

$= \log_e y$

4 **(b)**

Let a and b be the same first and last terms of the three progressions, each having $(2n+1)$ terms. Then,

The middle term of the A.P. $= \frac{a+b}{2}$

The middle term of the G.P. $= \sqrt{ab}$

The middle term of the H.P. $= \frac{2\,ab}{a+b}$

Obviously, these terms are in G.P.

5 **(b)**

Let $S = \frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + \ldots$

$$\therefore T_n = \frac{1.3.5.....(2n-1)}{1.2.3.....(2n-1)2n} \times \frac{(2.4.8.....2n)}{(2.4.8.....2n)}$$

$$= \frac{(2n)!}{(2n)!.2^n(n!)} = \frac{1}{2^n(n!)}$$

$$\therefore S = \sum T_n = \frac{1}{2.1!} + \frac{1}{2^2.2!} + \frac{1}{2^3.3!} + \ldots$$

$$= e^{1/2} - 1$$

6 **(c)**

We know that, arithmetic mean of a and $b = \frac{a+b}{2}$.

But given that $\frac{a+b}{2} = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$

$$\Rightarrow a^n + b^n + \frac{ab^n}{b} + \frac{ba^n}{a} = 2(a^n + b^n)$$

$$\Rightarrow \frac{a}{b}b^n + \frac{b}{a}a^n = a^n + b^n$$

$$\Rightarrow a^n\left(\frac{a-b}{a}\right) = -b^n\left(\frac{b-a}{b}\right)$$

$$\Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)$$

$$\therefore n = 1$$

7 **(d)**

$0.14189189189\ldots$

$$= 0.14 + 0.00189 + 0.00000189 + \ldots$$

$$= \frac{14}{100} + 189\left[\frac{1}{10^5} + \frac{1}{10^8} + \ldots\infty\right]$$

$$= \frac{7}{50} + \frac{189}{999 \times 100}$$

$$= \frac{7}{50} + \frac{7}{3700} = \frac{21}{148}$$

8 **(c)**

The series is $\log_{3^2} 3 + \log_{3^3} 3 - \log_{3^4} 3 + \log_{3^5} 3 - \ldots$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots + 1 - 1$$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$$

$$= \log_e(1+1) = \log_e 2$$

9 **(a)**

Since, $2b = a + c$

Now, $(a + 2b - c)(2b + c - a)(a + 2b + c)$

$$= (a + a + c - c)(a + c + c - a)(2b + 2b)$$

$$= 2a.2c.4b = 16abc$$

10 **(a)**

$1 \cdot n + 2(n-1) + 3(n-2) + \ldots + n \cdot 1$

$= \sum_{r=1}^{n} (n+1)r - \sum_{r=1}^{n} r^2$

$= (n+1)\sum n - \sum n^2$

$= \frac{(n+1)n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)}{6}\{3n+3-2n-1\}$

$= \frac{n(n+1)(n+2)}{6}$

11 **(b)**

We have,

$a^x = b^y = c^z = d^\omega$

$\Rightarrow a^x = b^y, a^x = c^z$ and $a^x = d^\omega$

$\Rightarrow x \log a = y \log b, x \log a = z \log c$ and $x \log a = \omega \log d$

$\Rightarrow \frac{x}{y} = \log_a b, \frac{x}{z} = \log_a c$ and $\frac{x}{\omega} = \log_a d$

$\Rightarrow \frac{x}{y} + \frac{x}{z} + \frac{x}{\omega} = \log_a b + \log_a c + \log_a d$

$\Rightarrow x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{\omega}\right) = \log_a bcd$

12 **(c)**

We have,

$2^{\frac{3}{\log_3 x}} = \frac{1}{64} \Rightarrow 2^{\frac{3}{\log_3 x}} = 2^{-6} \Rightarrow \frac{3}{\log_3 x} = -6$

$\Rightarrow \log_3 x = -\frac{1}{2} \Rightarrow x = 3^{-1/2} = \frac{1}{\sqrt{3}}$

13 **(d)**

Let $S = 1 + 2 + 3 + ... + 100$

$= \frac{100}{2}(1+100) = 50(101) = 5050$

Let $S_1 = 3 + 6 + 9 + 12 + ... + 99$

$= 3(1 + 2 + 3 + 4 + ... + 33)$

$= 3.\frac{33}{2}(1+33) = 99 \times 17 = 1683$

Let $S_2 = 5 + 10 + 15 + ... + 100$

$= 5(1 + 2 + 3 + ... + 20)$

$= 5.\frac{20}{2}(1+20) = 50 \times 21 = 1050$

Let $S_3 = 15 + 30 + 45 + ... + 90$

$= 15(1 + 2 + 3 + ... + 6)$

$= 15.\frac{6}{2}(1+6) = 45 \times 7 = 315$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632$

14 **(d)**

Given that, $T_3 = ar^2 = P$

Let first five terms of GP series be a, ar, ar^2, ar^3, ar^4

Now, $a.ar.ar^2.ar^3.ar^4 = a^5r^{10} = \left(ar^2\right)^5 = P^5$

15 **(c)**

Let A be the first term and D be the common difference of the AP. Then,

$S_n = an(n-1)$

$\Rightarrow \frac{n}{2}\{2A + (n-1)D\} = an\,(n-1)$

$\Rightarrow 2A + (n-1)D = 2\,an - 2\,a$

$\Rightarrow 2A - D = -2\,a$ and $D = 2\,a$

$\Rightarrow A = 0, D = 2\,a$

The sum of the squares of the n terms of the sequence is

$S = A^2 + (A+D)^2 + (A+2D)^2 + \dots + \{A + (n-1)D\}^2$

$\Rightarrow S = D^2\{1^2 + 2^2 + 3^2 + \dots + (n-1)^2\}$

$\Rightarrow S = 4\,a^2 \frac{n(n-1)(2n-1)}{6} = \frac{2\,a^2}{3} n(n-1)(2n-1)$

16 **(a)**

We have,

$\log_{30} 8 = \log_{30} 2^3 = 3\log_{30} 2 = 3\log_{30}\left(\frac{30}{15}\right)$

$= 3(\log_{30} 30 - \log_{30} 15)$

$= 3\left(1 - \log_{10} 3 - \log_{30} 5\right) = 3(1 - x - y)$

17 **(c)**

We have,

$3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}} = 3^{\frac{4}{\log_2 3}} + 27^{\frac{1}{\log_6 3}} + 81^{\frac{1}{\log_5 3}}$

$= 3^{4\log_3 2} + 27^{\log_3 6} + 81^{\log_3 5} = 3^{\log_3 2^4} + \left(3^3\right)^{\log_3 6} + \left(3^4\right)^{\log_3 5}$

$= 3^{\log_3 16} + \left(3^3\right)^{\log_3 6} + \left(3^4\right)^{\log_3 5} = 3^{\log_3 16} + 3^{\log_3 6^3} + 3^{\log_3 5^4}$

$= 16 + 6^3 + 5^4 = 16 + 216 + 625 = 857$

18 **(a)**

Let $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = d$

$\therefore \frac{1}{d}\left(\frac{d}{a_1a_2} + \frac{d}{a_2a_3} + \dots + \frac{d}{a_{n-1}a_n}\right)$

$= \frac{1}{d}\left[\frac{a_2 - a_1}{a_1a_2} + \frac{a_3 - a_2}{a_2a_3} + \dots + \frac{a_n - a_{n-1}}{a_{n-1}a_n}\right]$

$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \dots + \frac{1}{a_{n-1}} - \frac{1}{a_n}\right]$

$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n - a_1}{a_1a_n}\right]$

$$= \frac{n-1}{a_1 a_n}$$

19 **(d)**

We have,

$$S = \sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)\,!}$$

$$\Rightarrow S = \sum_{n=2}^{\infty} \frac{n\,!}{(n-2)\,!(n+1)\,!2\,!}$$

$$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty} \frac{n(n-1)}{(n+1)\,!}$$

$$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty} \left(\frac{n^2-1-n-1+2}{(n+1)\,!}\right)$$

$$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty} \left(\frac{n-1}{n\,!} - \frac{1}{n\,!} + \frac{2}{(n+1)\,!}\right)$$

$$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty} \left(\frac{1}{(n-1)\,!} - \frac{2}{n\,!} + \frac{2}{(n+1)\,!}\right)$$

$$\Rightarrow S = \frac{1}{2}\left\{(e-1) - 2(e-2) + 2\left(e - \frac{5}{2}\right)\right\} = \frac{e}{2} - 1$$

20 **(b)**

Since a, b,c are in H.P. Therefore,

$$b = \frac{2ac}{a+c}$$

$$\therefore \frac{1}{\frac{2ac}{a+c} - a} + \frac{1}{\frac{2ac}{a+c} - c}$$

$$= \frac{a+c}{ac-a^2} + \frac{a+c}{ac-c^2}$$

$$= \frac{a+c}{a(c-a)} + \frac{a+c}{c(a-c)} = \frac{(a+c)(c-a)}{ac(c-a)} = \frac{1}{a} + \frac{1}{c}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	B	B	C	D	C	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	D	C	A	C	A	D	B