DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :8

Topic :-SEQUENCES AND SERIES

1 **(a)**

Here, T_n of the AP 1,2,3... $= n$

and T_n of the AP 3,5,7... $= 2n+1$

$\therefore T_n$ of given series $= n(2n+1)^2 = 4n^3 + 4n^2 + n$

Hence, $S = \sum_{n=1}^{20} T_n = 4\sum_{n=1}^{20} n^3 + 4\sum_{n=1}^{20} n^2 + \sum_{n=1}^{20} n$

$= 4\frac{1}{4}20^2.21^2 + 4\frac{1}{6}20.21.41 + \frac{1}{2}20.21$

$= 188090$

2 **(c)**

$$\frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a} \quad [T_2 - T_1 = T_3 - T_2]$$

$$\Rightarrow \frac{b^2+bc-ac-a^2}{(c+a)(b+c)} = \frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$$

$\Rightarrow \{b^2-a^2-c(a-b)\}(a+b) = \{c^2-b^2-a(b-c)\}(b+c)$

$\Rightarrow (b^2-a^2)(b+a+c) = (c^2-b^2)(a+b+c)$

$\Rightarrow 2b^2 = a^2+c^2$

$\Rightarrow a^2, b^2, c^2$ are in AP

3 **(a)**

Since, $\frac{(S_n)_1}{(S_n)_2} = \frac{2n+3}{6n+5}$ ...(i)

$$\Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{2n+3}{6n+5}$$

Put $\frac{n-1}{2} = 12 \Rightarrow n = 25$

$$\therefore \frac{a_1+12d_1}{a_2+12d_2} = \frac{53}{155}$$

$\Rightarrow \dfrac{(T_{13})_1}{(T_{13})_2} = \dfrac{53}{155}$

4 **(c)**

It is given that

$\dfrac{x+y}{1-xy}, y, \dfrac{y+z}{1-yz}$ are in A.P.

$\Rightarrow y - \dfrac{x+y}{1-xy} = \dfrac{y+z}{1-yz} - y$

$\Rightarrow \dfrac{y - xy^2 - x - y}{1-xy} = \dfrac{y+z-y+y^2z}{1-yz}$

$\Rightarrow -\dfrac{x}{1-xy} = \dfrac{z}{1-yz}$

$\Rightarrow -x + xyz = z - xyz$

$\Rightarrow 2\,xyz = x + z$

$\Rightarrow y = \dfrac{x+z}{2\,xz} \Rightarrow \dfrac{1}{y} = \dfrac{2\,xz}{x+z} \Rightarrow x, \dfrac{1}{y}, z$ are in H.P.

5 **(a)**

We have, $x + y + z = 15$, if $9, x, y, z, a$ are in AP.

$\therefore$ Sum $= 9 + 15 + a = \frac{5}{2}(9 + a)$

$\Rightarrow 24 + a = \dfrac{5}{2}(9 + a)$

$\Rightarrow 48 + 2a = 45 + 5a$

$\Rightarrow 3a = 3 \Rightarrow a = 1$ ...(i)

And $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{5}{3}$, if $9, x, y, z, a$ are in HP.

Sum $= \dfrac{1}{9} + \dfrac{5}{3} + \dfrac{1}{a} = \dfrac{5}{2}\left[\dfrac{1}{9} + \dfrac{1}{a}\right] \Rightarrow a = 1$

6 **(d)**

Since, a, b, c are in AP.

(A) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP $\Rightarrow \frac{k}{a}, \frac{k}{b}, \frac{k}{c}$ are in HP.

(B) $a + k, b + k, c + k$ are in AP.

(C) ka, kb, kc are in AP.

(D) a^2, b^2, c^2 are in AP.

Then, $b^2 - a^2 = c^2 - b^2$

$\therefore (b - a)(b + a) = (c - b)(c + b) = (b - a)(c + b)$

$\left[\begin{array}{c} \because a, b, c \text{ are in AP} \\ \therefore b - a = c - b \end{array}\right]$

$\Rightarrow b + a = c + b$

$\Rightarrow a = c$, which is not true

7 **(d)**

We have,

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$$

$$\therefore \log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)=\log_{2.5}^{\left(\frac{1}{2}\right)}=\log_{\left(\frac{2}{5}\right)}-1\,(2)^{-1}$$

$$=\log_{(2/5)}2=\log_{0.4}2$$

$$\therefore (0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}$$

$$=(0.16)^{\log_{0.4}2}$$

$$=\{(0.4)^2\}^{\log_{0.4}2}=(0.4)^{2\log_{0.4}2}=(0.4)^{\log_{0.4}2^2}=2^2=4$$

8 **(a)**

We have,

$$\frac{3+5+7+\ldots+n\text{ (terms)}}{5+8+11+\ldots+10\text{ terms}}=7$$

$$\Rightarrow\frac{\frac{n}{2}\{6+(n-1)2\}}{\frac{10}{2}\{10+(10-1)3\}}=7$$

$$\Rightarrow\frac{n(n+2)}{5\times 37}=7$$

$$\Rightarrow n^2+2n=35\times 37\Rightarrow(n+37)(n-35)=0\Rightarrow n=35$$

10 **(c)**

Let $a_2=ra_1, a_3=r^2a_1,\ldots$, so on

$$\therefore\ \frac{a_1-a_3+a_5-\ldots+a_{49}}{a_2-a_4+a_6-\ldots+a_{50}}$$

$$=\frac{a_1-r^2a_1+r^4a_1-\ldots+r^{48}a_1}{a_1r-r^3a_1+r^5a_1-\ldots+r^{49}a_1}$$

$$=\frac{a_1(1-r^2+r^4-\ldots+r^{48})}{a_1r(1-r^2+r^4-\ldots+r^{48})}$$

$$=\frac{1}{r}=\frac{a_1}{a_2}$$

11 **(c)**

As a, b, c, d are in HP. So, b is HM between a and c. Also, GM between a and $c=\sqrt{ac}$.

Now, GM > HM

$\Rightarrow\sqrt{ac}>b$

$\Rightarrow ac>b^2$ ...(i)

Again, a, b, c, d are in HP. So c is HM between b and d.

Therefore, $bd>c^2$ ...(ii)

On multiplying relations (i) and (ii), we get

$abcd>b^2c^2\Rightarrow ad>bc$

Hence, option (b) is true.

Now, AM between a and c $=\frac{1}{2}(a+c)$
Now, as AM > HM
So, here $a+c>2b$ ...(iii)
And c is HM between b and d
$\Rightarrow b+d>2c$ ...(iv)
On adding relations (iii) and (iv), we get
$(a+c)+(b+d)>2(b+c)$
$\Rightarrow a+d>b+c$
So, both (a) and (b) are correct.

12 **(d)**

The sum to infinity of the given G.P. exists, iff.
$\left|\frac{3}{x}\right|<1 \Leftrightarrow |x|>3$

14 **(d)**

We have,
$9a^2+4b^2=18ab$
$\Rightarrow 9a^2+12ab+4b^2=30ab$
$\Rightarrow (3a+2b)^2=30ab$
$\Rightarrow 2\log(3a+2b)= \log(5a\times 3b\times 2)$
$\Rightarrow \log(3a+2b)=\frac{1}{2}\{\log 5a+\log 3b+\log 2\}$

15 **(d)**

We have,
$$S=\sum_{n=1}^{\infty}\left\{\frac{{}^nC_0+{}^nC_1+{}^nC_2+\ldots+{}^nC_n}{{}^nP_n}\right\}=\sum_{n=1}^{\infty}\frac{2^n}{n\,!}=e^2-1$$

16 **(a)**

The number of divisors of 3×7^3=(1+1)(3+1)=8
The number of divisors of $7\times 11^2=(1+1)(2+1)=6$
And the number of divisors of $2\times 61=(1+1)(1+1)=4$
$\Rightarrow$8, 6, 4 are in AP with common difference-2

17 **(b)**

We have,
$\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$
$\Rightarrow \frac{1}{a}+\frac{1}{c-b}=\frac{1}{b-a}-\frac{1}{c}$
$\Rightarrow \frac{a+c-b}{a(c-b)}=\frac{c-b+a}{c(b-a)}$
$\Rightarrow a(c-b)=c(b-a)$
$\Rightarrow ac-ab=bc-ac$
$\Rightarrow 2ac=ab+bc \Rightarrow \frac{2ac}{a+c}=b \Rightarrow a,b,c$ are in H.P.

18 **(b)**

Let four numbers are $a-3d, a-d, a+d, a+3d$.
$\therefore (a-3d)+(a+3d)=8$
$\Rightarrow (a-d)(a+d)=15$
and $(a-d)(a+d)=15$
$\Rightarrow a^2-d^2=15$
$\Rightarrow d=1$
Thus, required numbers are 1, 3, 5, 7 .
Hence, greatest number is 7.
19 **(d)**
We have,
$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\dots+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$$
$$=(\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\dots+(\sqrt{n^2}-\sqrt{n^2-1})$$
$$=\sqrt{n^2}-1=n-1$$
20 **(b)**
Using A.M. $> G.M.$, we have
$$\left.\begin{array}{l} x+y>2\sqrt{xy} \\ y+z>2\sqrt{yz} \\ y+x>2\sqrt{xz} \end{array}\right\} \Rightarrow (x+y)(y+z)(z+x)>8\,xyz$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	C	A	D	D	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	A	D	D	A	B	B	D	B