DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :7

Topic :-SEQUENCES AND SERIES

1 **(c)**

Since, a, b, c, d, e, f are in AP.

So, $b - a = c - b = d - c = e - d = f - e = k$

Where k is the common difference

Now, $d - c = e - d \Rightarrow e + c = 2d$

$\Rightarrow e - c + 2c = 2d \Rightarrow e - c = 2(d - c)$

2 **(b)**

Let A and R be the first term and common ratio of the GP, then

$a = AR^{p-1}, b = AR^{q-1}$ and $= AR^{r-1}$ ...(i)

Again, if x and d be the first term and common difference of an AP corresponding to the given HP, then

$\frac{1}{a} = x + (p-1)d,\ \frac{1}{b} = x + (q-1)d,\ \frac{1}{c} = x + (r-1)d$ ...(ii)

From Eq.(i), $\frac{a}{b} = R^{p-q}$

$\Rightarrow \left(\frac{a}{b}\right)^{1/c} = \left(R^{p-q}\right)^{1/c} = R^k,$

Where $k = \frac{p-q}{c} = (p-q)\{x + (r-1)d\}$ [from Eq. (ii)]

$= (p-q)x + (p-q)(r-1)d$

$= (p-q)x - (p-q)d + (rp - rq)d$ ...(iii)

Similarly, $\left(\frac{b}{c}\right)^{1/a} = \left(R^{q-r}\right)^{1/a} = R^n,$

Where $n = \frac{(q-r)}{a} = (q-r) \times \{x + (p-1)d\}$ [from Eq.(ii)]

$\Rightarrow n = (q-r)x - (q-r)d + (pq - pr)d$ ...(iv)

And $\left(\frac{c}{a}\right)^{1/b} = \left(R^{r-p}\right)^{1/b} = R^m$

Where $m = \frac{r-p}{b} = (r-p)\{x + (q-1)d\}$ [from Eq.(ii)]

$= (r-p)x(r-p)d + (rq - qp)d$ ...(v)

$\therefore \left(\frac{a}{b}\right)^{1/c}\left(\frac{b}{c}\right)^{1/a}\left(\frac{c}{a}\right)^{1/b} = R^k R^m R^n = R^{m+n+k} = R^0 = 1$

[Since, $k + m + n = 0$, adding Eqs. (iii), (iv) and (v)]

Taking log on both sides, we get

$\frac{1}{c}(\log a - \log b) + \frac{1}{a}(\log b - \log c) + \frac{1}{b}(\log c - \log a) = 1 \log(1)$

$\Rightarrow \left(\frac{1}{c} - \frac{1}{b}\right)\log a + \left(\frac{1}{a} - \frac{1}{c}\right)\log b + \left(\frac{1}{b} - \frac{1}{a}\right)\log c = 0$

$\Rightarrow\left(\frac{b-c}{bc}\right)\log a+\left(\frac{c-a}{ac}\right)\log b+\left(\frac{a-b}{ab}\right)\log c=0$

$\Rightarrow a(b-c)\log a+b(c-a)\log b+c(a-b)\log c=0$

3 **(a)**

Here, $T_n=\sum_{n=1}^{\infty}\frac{1}{(n+a)(n+1+a)}$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{n+a}-\frac{1}{n+1+a}\right)$$

$$\therefore S_n=\Sigma T_n=\left(\frac{1}{1+a}-\frac{1}{2+a}\right)+\left(\frac{1}{2+a}-\frac{1}{3+a}\right)$$

$$+\ldots+\left(\frac{1}{n+a}-\frac{1}{n+1+a}\right)$$

$$\Rightarrow S_n=\frac{1}{1+a}-\frac{1}{n+1+a}$$

$$\Rightarrow \lim_{n\to\infty} S_n=\frac{1}{1+a}$$

4 **(b)**

The two sides of the equation are meaningful, if $-x>0$ and $x+1>0$ i.e. if $x\in(-1,0)$

Now,

$\log(-x)\ =2\log(x+1)$

$\Rightarrow -x=(x+1)^2$

$$\Rightarrow x^2+3x+1=0\Rightarrow x=\frac{-3+\sqrt{5}}{2}\qquad [\because x\in(-1,0)]$$

5 **(b)**

Let $S=0.1\dot{2}\dot{3}$. Then,

$S=0.42323232323\ldots$

$\Rightarrow S=0.4+0.023+0.00023+\ldots$

$\Rightarrow S=0.4+23\times10^{-3}+23\times10^{-5}+\ldots$

$$\Rightarrow S=0.4+\frac{23\times10^{-3}}{1-10^{-2}}=0.4+\frac{23}{990}=\frac{419}{990}$$

6 **(b)**

$$\sum_{r=1}^{n}\sum_{s=1}^{n}S_{rs}2^r3^s=2\cdot3+2^2\cdot3^2+2^3\cdot3^3+\ldots+2^n\cdot3^n$$

(as $S_{rs}=0$, if $r\neq s$ and $S_{rs}=1$, if $r=s$)

$$=\frac{6(6^n-1)}{6-1}=\frac{6}{5}(6^n-1)$$

7 **(b)**

We have,

$$2b=a+c,\ d=\frac{2ce}{c+e}\text{ and }c^2=bd$$

On eliminating b and d, we obtain

$c^2=ae\Rightarrow a,c,e$ are in G.P.

8 **(a)**

$$2\left[\frac{1}{7}+\frac{1}{3}\cdot\frac{1}{7^3}+\frac{1}{5}\cdot\frac{1}{7^5}+\dots\right]=\log_e\left[\frac{1+1/7}{1-1/7}\right]=\log_e\frac{4}{3}$$

9 **(d)**

$t_{11}+t_{12}+t_{13}=141$

And $t_{21}+t_{22}+t_{23}=261$

$\therefore 3a+33d=141$

$\Rightarrow a+11d=47$ ...(i)

And $3a+63d=261$

$\Rightarrow a+21d=87$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a=3, \quad d=4$

10 **(c)**

We have,

2^{n+10}

$=2\times2^2+3\times2^3+4\times2^4+\dots+(n-1)\times2^{n-1}+n\times2^n$ ...(i)

$\Rightarrow 2\times2^{n+10}$

$=2\times2^3+3\times2^4+\dots+(n-1)2^n+n\times2^{n+1}$ ...(ii)

Subtracting (ii) from (i), we get

$-2^{n+10}=2\times2^2+(2^3+2^4+\dots+2^n)-n\times2^{n+1}$

$\Rightarrow -2^{n+10}=8+8(2^{n-2}-1)-n\times2^{n+1}$

$-2^{n+10}=2^{n+1}-n\times2^{n+1}$

$\Rightarrow -2^{10}=2-2n \Rightarrow n=513$

11 **(c)**

As we know, sum infinite terms of GP,

$$S_\infty=\begin{cases}\frac{a}{1-r}, & |r|<1\\ \infty, & |r|\geq 1\end{cases}$$

$\therefore \; S_\infty=\frac{x}{1-r}=5 \qquad \{\text{thus } |r|<1\}$

$\Rightarrow 1-r=\frac{x}{5}$

$\Rightarrow r=\frac{5-x}{5}$ exists only when $|r|<1$

$\Rightarrow -1<\frac{5-x}{5}<1$

$\therefore \; -10<-x<0 \Rightarrow 0<x<10$

12 **(c)**

For $x=-2$, we have

$\log_4\left(\frac{x^2}{4}\right) - 2\log_4(4x^4)$

$= \log_4 1 - 2\log_{2^2}(2^6) = 0 - 2 \times \frac{6}{2}\log_2 2 = -6$

13 **(c)**

We have,

$\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x} = \lambda(\text{say})$

$\Rightarrow \log 3 = \lambda(x-y), \log 5 = \lambda(y-z), \log 7 = \lambda(z-x)$

$\Rightarrow 3 = 10^{\lambda(x-y)}, 5 = 10^{\lambda(y-z)}, 7 = 10^{\lambda(z-x)}$

$\Rightarrow 3^{x+y}.5^{z+x}.7^{z+x} = 10^{\lambda(x^2-y^2)}.10^{\lambda(y^2-z^2)}.10^{\lambda(z^2-x^2)}$

$\Rightarrow 3^{x+y}.5^{y+z}.7^{z+x} = 10^{\lambda(x^2-y^2+y^2-z^2+z^2-x^2)} = 10^0 = 1$

14 **(d)**

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i} j$$

$$= \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2}\sum_{i=1}^{n} (i^2 + i)$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}(2n+4)$$

$$= \frac{n(n+1)(n+2)}{6}$$

15 **(b)**

We have,

$\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \ldots\text{to } \infty$

$= \sum_{n=1}^{\infty} \frac{2n}{(2n+1)} = \sum_{n=1}^{\infty} \frac{(2n+1)}{(2n+1)!} = \sum_{n=1}^{\infty} \left\{\frac{1}{2n!} - \frac{1}{(2n+1)!}\right\}$

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \ldots\text{to } \infty$

$= e^{-1}$

16 **(b)**

Let $S_n = \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots + \frac{1}{5n+1} - \frac{1}{5n+6}\right)$

$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots + \frac{1}{5n+1} - \frac{1}{5n+6}\right)$

$$=\frac{1}{5}\left(\frac{1}{6}-\frac{1}{5n+6}\right)=\frac{n}{6(5n+6)}$$

$$\Rightarrow 6S_n=\frac{n}{5n+6}$$

17 **(c)**

We have,

$$\log(x-y)-\log 5-\frac{1}{2}\log x-\frac{1}{2}\log y=0$$

$$\Rightarrow 2\log(x-y)-2\log 5-\log x-\log y=0$$

$$\Rightarrow \frac{(x-y)^2}{25xy}=1 \Rightarrow \left(\frac{x-y}{\sqrt{xy}}\right)^2=25$$

$$\Rightarrow \left(\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}\right)^2=25 \Rightarrow \frac{x}{y}+\frac{y}{x}-2=25 \Rightarrow \frac{x}{y}+\frac{y}{x}=27$$

18 **(b)**

It is given that

$\log_a x, \log_b x, \log_c x$ are in A.P.

$\Rightarrow 2\log_b x = \log_a x+\log_c x$

$$\Rightarrow \frac{2\log x}{\log b}=\frac{\log x}{\log a}+\frac{\log x}{\log c}$$

$$\Rightarrow \frac{2\log a\log c}{\log b}=\log a+\log c$$

$$\Rightarrow \frac{2\log a\log c}{\log b}=\log(ac)$$

$\Rightarrow 2\log c\log_b a = \log ac$

$\Rightarrow \log c^{2\log_b a} = \log ac$

$\Rightarrow c^{2\log_b a}=ac \Rightarrow (c^2)=(ac)^{\log_a b}$

19 **(a)**

Let $a^{1/x}=b^{1/y}=c^{1/z}=k$ [say]

$\Rightarrow \log a = x\log k$, $\log b = y\log k$

and $\log c = z\log k$

Since, $b^2=ac$

$\Rightarrow 2\log b = \log a+\log c$

$\Rightarrow 2(y\log k)=x\log k+z\log k$

$\Rightarrow 2y=x+z$

$\Rightarrow x,y,z$ are in AP.

20 **(a)**

$$(1-2x-x^2)(e^x)$$

$$=(1-2x-x^2)\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots\infty\right)$$

$$=\left(1+x+\frac{x^2}{2!}+\ldots+\frac{x^k}{k!}+\ldots\infty\right)$$

$$-2\left(x+x^2+\frac{x^3}{2!}+\ldots+\frac{x^k}{(k-1)!}+\frac{x^{k+1}}{k!}+\ldots\infty\right)$$

$$-\left(x^2+x^3+\frac{x^4}{2!}+\ldots\frac{x^k}{(k-2)!}+\frac{x^{k+1}}{(k-1)!}+\frac{x^{k+2}}{k!}+\ldots\infty\right)$$

$\therefore$ Cofficient of x^kin $\left(\frac{1-2x-x^2}{e^{-x}}\right)=\frac{1}{k!}-\frac{2}{(k-1)!}-\frac{1}{(k-2)!}$

$$=\frac{1}{k!}-\frac{2k}{k!}-\frac{k(k-1)}{k!}$$

$$=\frac{1-k-k^2}{k!}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	A	B	B	B	B	A	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	D	B	B	C	B	A	A