DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :6

Topic :-SEQUENCES AND SERIES

1 **(b)**

If x, y, z are in HP, then

$y=\frac{2xz}{x+z}$ ...(i)

Now, $\log(x+z)+\log(x-2y+z)$

$=\log[(x+z)(x-2y+z)]$

$=\log\left[(x+z)\left(x+z-\frac{4xz}{x+z}\right)\right]$ [from Eq.(i)]

$=\log\left[(x+z)^2-4xz\right]$

$=\log(x-z)^2$

$=2\log(x-z)$

2 **(c)**

Let a be the first term and d be the common difference of the A.P. Then,

$a_m=a+(m-1)d$ ...(i)

$a_n=a+(n-1)d$ ...(ii)

$a_p=a+(p-1)d$ ...(iii)

Multiplying (i), (ii) and (iii) respectively by $(n-p)$,$(p-m)$ and $(m-n)$ and adding, we get

$a_m(n-p)+a_n(p-m)+a_p(m-n)=0$ ...(iv)

Expanding along first row, we have

$\Delta=a_m(n-p)+a_n(p-m)+a_p(m-n)$

$\Rightarrow\Delta=0$ [Using (iv)]

3 **(c)**

Let$S=\frac{1}{n!}\left[\frac{n!}{1!(n-1)!}+\frac{n!}{3!(n-3)!}+\frac{n!}{5!(n-5)!}+\dots\right]$

$=\frac{1}{n!}[{}^nC_1+{}^nC_3+{}^nC_5+\dots]$

$=\frac{1}{n!}2^{n-1}$for all value of n only

4 **(b)**

The given equation is meaningful if $x-1>0$ and $x-3>0$ i.e. $x>3$

Now,

$\log_4(x-1)=\log_2(x-3)$

$\Rightarrow\frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\Rightarrow\log_2(x-1)=2\log_2(x-3)$

$\Rightarrow \log_2(x-1) = \log_2 (x-3)^2$
$\Rightarrow x-1 = (x-3)^2$
$\Rightarrow x^2 - 7x + 10 = 0 \Rightarrow x = 5,2 \Rightarrow x = 5 \quad [\because x > 3]$
Hence, the given equation has just one solution
55 **(a)**
Let T_r be the rth term of the given series. Then,
$T_r = 1 + 2 + 2^2 + \dots + 2^r = 2^{r+1} - 1$

$$\therefore \text{Required sum} = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} (2^{r+1} - 1)$$

$$\Rightarrow \text{Required sum } = 2^2\left(\frac{2^n - 1}{2-1}\right) - n = 2^{n+2} - n - 4$$

6 **(c)**
We have,

$$1 + \frac{(\log_e n)^2}{2\,!} + \frac{(\log_e n)^4}{4\,!} + \dots \text{to } \infty$$

$$= \frac{e^{\log_e n} + e^{-\log_e n}}{2} = \frac{1}{2}(n + n^{-1})$$

7 **(a)**

$$\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \dots$$

$$= (\log_e 3)\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots\right)$$

$$= (\log_e 3) \log_e 2$$

9 **(c)**
Given that, $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)
And $T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)
Where a and b are the first term and common difference respectively.
On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{mn} \text{ and } d = \frac{1}{mn}$$

$$\therefore T_{mn} = a + (mn-1)d$$

$$= \frac{1}{m} + (mn-1)\frac{1}{mn} = 1$$

10 **(a)**
Since, $l = A + (n-1)d$

$$\therefore c = a + (n-1)(b-a)$$

$$\Rightarrow (n-1) = \frac{c-a}{b-a}$$

$$\Rightarrow n = \frac{b+c-2a}{b-a}$$

11 **(c)**

Given, $x_n = x_{n+1}\sqrt{2}$

$\therefore\ x_1 = x_2\sqrt{2}, x_2 = x_3\sqrt{2},\ldots,x_n = x_{n+1}\sqrt{2}$

On multiplying $x_1 = x_{n+1}(\sqrt{2})^n$

$\Rightarrow x_{n+1} = x/(\sqrt{2})^n$

Hence, $x_n = \frac{x_1}{(\sqrt{2})^{n-1}}$

Area of $S_n = x_n^2 = \frac{x_1^2}{2^{n-1}} < 1 \Rightarrow 2^{n-1} > x_1^2 \quad (\because x_1 = 10)$

$\therefore\ 2^{n-1} > 100$

But $2^7 > 100, 2^8 > 100$ etc.

$\therefore\ n - 1 = 7,8,9,\ldots \Rightarrow n = 8, 9, 10,\ldots$

12 **(a)**

We have,

$$x = \frac{1}{1-a}, y = \frac{1}{1-b}$$

$$\Rightarrow a = 1 - \frac{1}{x}, b = 1 - \frac{1}{y}$$

$$\Rightarrow a = \frac{x-1}{x}, b = \frac{y-1}{y}$$

$$\therefore 1 + ab + a^2b^2 + \ldots = \frac{1}{1-ab} = \frac{xy}{x+y-1}$$

13 **(c)**

Given, $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$

$$\therefore\ (\sin d)[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \ldots + \sec a_{n-1} \sec a_n]$$

$$= \frac{\sin d}{\cos a_1 \cos a_2} + \frac{\sin d}{\cos a_2 \cos a_3} + \ldots + \frac{\sin d}{\cos a_{n-1} \cos a_n}$$

$$= \frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2} + \frac{\sin(a_3 - a_2)}{\cos a_2 \cos a_3} + \ldots + \frac{\sin(a_n - a_{n-1})}{\cos a_{n-1} \cos a_n}$$

$$= \tan a_2 - \tan a_1 + \tan a_3 - \tan a_2 + \ldots + \tan a_n - \tan a_{n-1}$$

$$= \tan a_n - \tan a_1$$

15 **(c)**

Since, $a_1, a_2, a_3, \ldots, a_n$ are in HP.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n}$ are in AP.

Let d be common difference of AP, then $\frac{1}{a_2} - \frac{1}{a_1} = d$

$\Rightarrow a_1 - a_2 = a_1 a_2 d$

Similarly, $a_2 - a_3 = a_2 a_3 d$

… …. …. ….

… …. …. ….

$a_{n-1} - a_n = a_{n-1} a_n d$

On adding all of these, we get

$a_1 - a_n = d(a_1a_2 + a_2a_3 + ... + a_{n-1}a_n)$ ...(i)

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$\Rightarrow d = \frac{a_1 - a_2}{a_1a_n(n-1)}$

On putting the value of d in Eq. (i), we get

$a_1 - a_n = \frac{a_1 - a_n}{a_1a_n(n-1)}(a_1a_2 + a_2a_3 + ... + a_{n-1}a_n)$

$\Rightarrow a_1a_2 + a_2a_3 + ... + a_{n-1}a_n = a_1a_n(n-1)$

17 **(b)**

We have,

$2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10} 2}$

$\Rightarrow (3\sqrt{3})^{\log_{10} 2} = 3^{k \log_{10} 2}$ $[\because x^{\log_a y} = y^{\log_a x}]$

$\Rightarrow 3^{\frac{3}{2}\log_{10} 2} = 3^{k \log_{10} 2} \Rightarrow k = \frac{3}{2}$

18 **(b)**

Since, $\beta\beta\gamma\delta = 1$ ...(i)

As we know, AM $\geq$ GM

$\Rightarrow \frac{1+\alpha}{2} \geq \sqrt{\alpha} \Rightarrow 1 + \alpha \geq 2\sqrt{\alpha}$ .....(ii)

Similarly, $1 + \beta \geq 2\sqrt{\beta}$ ...(iii)

$1 + \gamma \geq 2\sqrt{\gamma}$ ...(iv)

And $1 + \delta \geq 2\sqrt{\delta}$ ...(v)

On multiplying Eqs. (ii), (iii), (iv) and (v), we get

$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16\sqrt{\alpha\beta\gamma\delta}$

Least value of $(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16$

19 **(b)**

Let $S = 6 + 66 + 666 + ...n$ terms

$= \frac{6}{9}(9 + 99 + 999 + ...n$ terms$)$

$= \frac{2}{3}[(10-1) + (100-1) + (1000-1) + ...n$ terms$]$

$= \frac{2}{3}\left[10 \cdot \frac{(10^n - 1)}{9} - n\right] = \frac{2}{27}[10^{n+1} - 10 - 9n]$

20 **(b)**

$\frac{4}{3} + \frac{10}{9} + \frac{28}{27} + ...$upto n terms

$= \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{9}\right) + \left(1 + \frac{1}{27}\right) + ...$upto n terms

$= (1 + 1 + 1 + ...n \text{ terms}) + \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...n \text{ terms}\right)$

$$= n + \frac{1}{3}\left(\frac{1 - \frac{1}{3^n}}{1 - \frac{1}{3}}\right) = \frac{3^n(2n+1) - 1}{2(3^n)}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	B	A	C	A	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	A	C	A	B	B	B	B