DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :5

Topic :-SEQUENCES AND SERIES

1 **(a)**

Since $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Therefore,

$(a^{x/2})^2 = \log_x a \cdot \log_b x \Rightarrow a^x = \log_b a \Rightarrow x = \log_a(\log_b a)$

2 **(a)**

Let

$S = i - 2 - 3i + 4 + 5i .... + 100 i^{100}$

$\Rightarrow S = i + 2i^2 + 3i^3 + 4i^4 + 5i^5 .... + 100 i^{100}$

$\Rightarrow i S = i^2 + 2i^3 + 3i^4 .... + 99 i^{100} + 100 i^{101}$

$\therefore S - iS = i + \{i^2 + i^3 + i^4 + ... + i^{100}\} - 100 i^{101}$

$S(1-i) = i + i^2\left\{\frac{(1-i^{99})}{(1-i)}\right\} - 100 i^{101}$

$\Rightarrow S(1-i) = i - \frac{(1+i)}{(1-i)} - 100i = i + 1 - 1 - i - 100i = -100i$

$\Rightarrow S = \frac{-100i}{1-i} = -50i(1+i) = -50(i-1) = 50(1-i)$

3 **(a)**

We have,

$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$

$\Rightarrow \frac{7}{4}\log_2 x = \frac{21}{4} \Rightarrow \log_2 x = 3 \Rightarrow x = 2^3 = 8$

4 **(c)**

The given numbers are 13, 17, … , 97. This is an AP with the first term 13 and common difference 4.

Let the number of term be n. Then

$97 = 13 + (n-1)4 \Rightarrow 4n = 88 \Rightarrow n = 22$

Therefore, the sum of the numbers

$S = \frac{n}{2}(a + l)$

$= \frac{22}{2}[13 + 97] = 11(110) = 1210$

5 **(c)**

Let $S_n = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)}$

$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + ... + \frac{1}{3n-1} - \frac{1}{3n+2}\right]$

$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{n}{6n+4}$

6 **(a)**

We have, $4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 8ac = 0$

$\Rightarrow 8a^2 + 18b^2 + 32c^2 - 12ab - 24bc - 16ac = 0$

$\Rightarrow 4a^2 + 9b^2 - 12ab + 9b^2 + 16c^2 - 24bc + 16c^2 + 4a^2 - 16ac = 0$

$\Rightarrow (2a - 3b)^2 + (3b - 4c)^2 + (4c - 2a)^2 = 0$

$\Rightarrow 2a = 3b = 4c = k$

$\Rightarrow a = \frac{k}{2}, b = \frac{k}{3}, c = \frac{k}{4}$

$\Rightarrow a, b, c$ are in HP

GM $\geq$ HM

$\therefore \sqrt{ac} \geq b$

7 **(c)**

Let A_j, H_j where $j = 1, 2, 3, ..., 9$ denote the 9 AM's and HM's between 2 and 3.

Then $2, A_1, A_2, A_3, ..., A_9, 3$ are in AP, let d be the common difference of this AP, then

$3 = 2 + 10d \Rightarrow d = \frac{1}{10}$

If A denotes the jth arithmetic mean, then

$A = 2 + jd = 2 + \left(\frac{j}{10}\right) \quad \left(\because d = \frac{1}{10}\right)$...(i)

Again, $2, H_1, H_2, ..., H_9, 3$ will be in HP.

$\Rightarrow \frac{1}{2}, \frac{1}{H_1}, \frac{1}{H_2}, ... \frac{1}{H_9}, \frac{1}{3}$ will be in AP

Let d be the common difference of this AP, then

$\frac{1}{3} = \frac{1}{2} + 10d \Rightarrow d = \frac{-1}{60}$

If H be the jth harmonic mean, then

$\frac{1}{H} = \frac{1}{2} + jd = \frac{1}{2} - \frac{j}{60}$ ...(ii)

$\therefore A + \frac{6}{H} = 2 + \frac{j}{10} + 6\left(\frac{1}{2} - \frac{j}{60}\right)$ [from Eqs.(i) and (ii)]

$= 5 + \frac{j}{10} - \frac{j}{10} = 5$

8 **(c)**

Since a, b, c are in G.P.

$\therefore b^2 = ac$

$\Rightarrow 2 \log_x b = \log_x a + \log_x c$

$\Rightarrow \frac{2}{\log_b x} = \frac{1}{\log_a x} + \frac{1}{\log_c x}$

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in H.P.

9 **(b)**

$\because$ 13, a_1, a_2,…,a_{20}, 67 are in AP

$$\therefore\ a_1 + a_2 + a_3 + \ldots + a_{20} = 20\left(\frac{13+67}{2}\right) = 800$$

Also, AM > GM

$$\Rightarrow \frac{a_1 + a_2 + a_3 + \ldots + a_{20}}{20} \geq (a_1\, a_2\, a_3 \ldots a_{20})^{1/20}$$

$\Rightarrow 40 \geq (a_1 \cdot a_2 \cdot a_3 \ldots, a_{20})^{1/20}$

Hence, maximum value of $a_1 \cdot a_2 \cdot a_3 \ldots, a_{20}$ is $(40)^{20}$

10 **(a)**

We have,

$$\log_{10}\left(\frac{n}{n-1}\right) = \log_e\left(\frac{n}{n-1}\right) \cdot \log_{10} e$$

$$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right) = -\log_e\left(\frac{n-1}{n}\right) \cdot \log_{10} e$$

$$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right) = -\log_{10} e \cdot \log_e\left(1-\frac{1}{n}\right)$$

$$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right) = \log_{10} e\left\{\sum_{r=1}^{\infty}\frac{1}{r}\left(\frac{1}{n}\right)^r\right\} = \sum_{r=1}^{\infty}\left\{\frac{1}{r}\log_{10} e\right\} n^{-r}$$

$\therefore$ Coefficient of $n^{-r} = \frac{1}{r}\log_{10} e = \frac{1}{r\log_e 10}$

11 **(c)**

We have,

$$\frac{1}{1-x} - \frac{1}{1+\sqrt{x}} = \frac{\sqrt{x}}{1-x}$$

and, $\frac{1}{1-\sqrt{x}} - \frac{1}{1-x} = \frac{\sqrt{x}}{1-x}$

hence, the terms are in A.P.

12 **(d)**

$$\left[(0.16)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\right)}\right]^{1/2}$$

$$= \left[(0.16)^{\log_{0.25}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)}\right]^{1/2}$$

$$= \left[(0.16)^{\log_{(0.5)^2} 0.5}\right]^{1/2}$$

$$= \left[(0.16)^{1/2}\right]^{1/2} = (0.4)^{1/2}$$

$= \frac{2}{\sqrt{10}}$

13 **(b)**

Number of notes that the person counts in 10 min

$= 10 \times 150 = 1500$

Since $a_{10}, a_{11}, a_{12}, \ldots$ are in AP with common difference -2

Let n be the time taken to count remaining 3000 notes, then

$\frac{n}{2}[2 \times 148 + (n-1) \times -2] = 3000$

$\Rightarrow \quad n^2 - 149n + 3000 = 0$

$\Rightarrow \quad (n-24)(n-125) = 0$

$\Rightarrow \quad n = 24, 125$

Then, the total time taken by the person to count all notes.

$= 10 + 24 = 34$ min

14 **(d)**

The given series is an A.P. with first term $a = 20$ and common diference $d = 2\frac{2}{3} = 8/3$

Let S_n denote the sum of n terms. Then,

$S_n > 1568$

$\Rightarrow \frac{n}{2}\left[40 + (n-1)\frac{8}{3}\right] > 1568$

$\Rightarrow n^2 + 14n - 1176 > 0$

$\Rightarrow (n+42)(n-28) > 0$

$\Rightarrow n > 28$

$\Rightarrow$ The least value of n is 29

15 **(b)**

Let a_p, a_q, a_r, a_s be $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ and s^{th} terms of the A.P. such that they are in G.P. with common ratio R.

$\therefore a_q = a_p R, a_r = a_p R^2$ and $a_s = a_p R^3$

$\Rightarrow a_q - a_p = a_p(R-1), a_r - a_q = a_p R(R-1), a_s - a_r = a_p R^2(R-1)$

$\Rightarrow a_q - a_p$, $a_r - a_q$, $a_s - a_r$ are in G.P.

$\Rightarrow a_p - a_q$, $a_q - a_r, a_r - a_s$ are in G.P.

$\Rightarrow (p-q)d$, $(q-r)d, (r-s)d$ are in G.P., where d is the common difference of the A.P.

$\Rightarrow p-q, q-r, r-s$ are in G.P.

16 **(a)**

Since, $2\tan^{-1} q = \tan^{-1} p + \tan^{-1} r$

$\Rightarrow \tan^{-1}\frac{2q}{1-q^2} = \tan^{-1}\frac{p+r}{1-p^r}$

$\Rightarrow \; 2q = p + r \qquad [\therefore q^2 = pr]$

$\Rightarrow \; p, q, r$ *are in* AP.

But p, q, r are in GP.

$\Rightarrow p = q = r$

18 **(c)**

It is given that $H_1, H_2, H_3, \ldots, H_n$ are n harmonic means between a and b. So, $a, H_1, H_2, H_3, \ldots, H_n, b$ are in HP

$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \ldots, \frac{1}{H_n}, \frac{1}{b}$ are in A.P. with common difference $D = \frac{a-b}{(n+1)ab}$

$$\therefore \frac{1}{H_1} = \frac{1}{a} + D \text{ and } \frac{1}{H_n} = \frac{1}{a} + n\,D$$

$$\Rightarrow \frac{1}{H_1} = \frac{1}{a} + \frac{a-b}{(n+1)ab} \text{ and } \frac{1}{H_n} = \frac{1}{a} + \frac{n(a-b)}{(n+1)\,ab}$$

$$\Rightarrow \frac{1}{H_1} = \frac{nb+a}{(n+1)\,ab} \text{ and } \frac{1}{H_n} = \frac{n\,a+b}{(n+1)\,ab}$$

$$\Rightarrow \frac{H_1}{a} = \frac{nb+b}{nb+a} \text{ and } \frac{H_n}{b} = \frac{na+a}{na+b}$$

$$\Rightarrow \frac{H_1+a}{H_1-a} = \frac{2nb+(a+b)}{b-a} \text{ and } \frac{H_n+b}{H_n-b} = \frac{2na+a+b}{a-b}$$

$$\Rightarrow \frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-b} = 2n$$

19 **(a)**

We have,

$$S_n = \frac{1}{2}\left\{\left(\sum_{r=1}^{n} r\right)^2 - \sum_{r=1}^{n} r^2\right\}$$

$$\Rightarrow S_n = \frac{1}{2}\left[\left\{\frac{n(n+1)}{2}\right\}^2 - \frac{n(n+1)(2n+1)}{6}\right]$$

$$\Rightarrow S_n = \frac{n(n^2-1)(3n+2)}{24}$$

$$\Rightarrow \frac{S_n}{(n+1)\,!} = \frac{1}{24}\left\{\frac{n(n^2-1)(3n+2)}{(n+1)\,!}\right\} = \frac{1}{24}\left\{\frac{3n+2}{(n-2)\,!}\right\}$$

$$\Rightarrow \frac{S_n}{(n+1)\,!} = \frac{1}{24}\left\{\frac{3(n-2)+8}{(n-2)\,!}\right\} = \frac{1}{8}\frac{1}{(n-3)\,!} + \frac{1}{3}\cdot\frac{1}{(n-2)\,!}$$

$$\therefore \sum_{n=0}^{\infty} \frac{S_n}{(n+1)\,!} = \frac{1}{8}\sum_{n=0}^{\infty}\frac{1}{(n-3)\,!} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{(n-2)\,!}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{S_n}{(n+1)\,!} = \frac{e}{8} + \frac{e}{3} = \frac{11e}{24}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	C	C	A	C	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	D	B	D	B	A	C	C	A	A