DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :4

Topic :-SEQUENCES AND SERIES

1 **(d)**

Let a, H_1, H_2, b are in HP.

$\therefore\ H_1 = \frac{3ab}{a+2b},\quad H_2 = \frac{3ab}{2a+b}$

Now, $\frac{H_1+H_2}{H_1H_2} = \frac{1}{H_1} + \frac{1}{H_2}$

$= \frac{2a+b}{3ab} + \frac{a+2b}{3ab} = \frac{a+b}{ab}$ ...(i)

Also, $2A = a + b$ ...(ii)

and $ab = G^2$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\frac{H_1+H_2}{H_1H_2} = \frac{2A}{G^2}$$

2 **(a)**

We have,

$$704\left\{1+\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^{n-1}}\right\}$$

$$= 1984\left\{1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\dots+\left(-\frac{1}{2}\right)^{n-1}\right\}$$

$$\Rightarrow 704\left\{\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right\} = 1984\left\{\frac{1-\left(-\frac{1}{2}\right)^n}{1+\frac{1}{2}}\right\}$$

$$\Rightarrow 128 = \frac{2112}{2^n} - \frac{1984(-1)^n}{2^n}$$

If n is odd, we get $2^n = 32 \Rightarrow n = 5$

If n is even, we get $128 = \frac{128}{2^n} \Rightarrow n = 0$

3 **(a)**

Let $S = 1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16+\dots}$

$$= 1 + \frac{(4-1)}{2} + \frac{(8-1)}{4} + \frac{(16-1)}{8} + \frac{(32-1)}{16} + \dots$$

$$= 1 + 2 - \frac{1}{2} + 2 - \frac{1}{4} + 2 - \frac{1}{8} + 2 - \frac{1}{16} + \dots$$

$$= 1 + 2(n-1) - \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + (n-1)\right]$$

$$1 + 2(n-1) - \left[\frac{\frac{1}{2}\left(1 - \frac{1}{2^{n-1}}\right)}{1 - \frac{1}{2}}\right]$$

$$= 1 + 2(n-1) - 1 + \frac{1}{2^{n-1}} = 2(n-1) + \frac{1}{2^{n-1}}$$

4 **(b)**

Since, $y = \frac{2xz}{x+z}$

Now, $x - 2y + z = x + z - 2\left(\frac{2xz}{x+z}\right)$

$$= x + z - \frac{4xz}{x+z} = \frac{(x-z)^2}{x+z}$$

$\Rightarrow \log(x - 2y + z) = \log(x-z)^2 - \log(x+z)$

$\Rightarrow \log(x - 2y + z) + \log(x+z) = 2\log(x-z)$

5 **(d)**

Since, $\frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5} = \frac{125}{162}$

$$\Rightarrow \frac{1 + r + r^2}{(1 + r + r^2)(1 + r^3)} = \frac{125}{162}$$

$$\Rightarrow 1 + r^3 = \frac{152}{125}$$

$$\Rightarrow r^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow r = \frac{3}{5}$$

6 **(c)**

Let T_r be the rth term of the given series. Then,

$$T_r = 1 + x + x^2 + \dots + x^{r-1} = \frac{1 - x^r}{1 - x}$$

$$\therefore \text{Required sum} = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \frac{1 - x^r}{1 - x} = \frac{1}{1-x}\sum_{r=1}^{n}(1 - x^r)$$

$\Rightarrow$Required sum $= \frac{1}{1-x}\left(\sum_{r=1}^{n} 1 - \sum_{r=1}^{n} x^r\right)$

$\Rightarrow$Required sum $= \frac{1}{1-x}\left\{n - x\left(\frac{1-x^n}{1-x}\right)\right\}$

$\Rightarrow$Required sum $= \frac{n(1-x) - x(1-x)^n}{(1-x)^2}$

7 **(b)**

Since, a, b, c are in GP.

$\Rightarrow b^2 = ac$

And $\log a - \log 2b, \log 2b - \log 3c$ and $\log 3c - \log a$ are in AP.

$\Rightarrow 2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a$

$\therefore\ b^2 = ac$ and $2b = 3c$

$\Rightarrow b = \frac{2a}{3}$ and $c = \frac{4a}{9}$

Since, $a + b = \frac{5a}{3} > c, b + c = \frac{10a}{9} > a,$

$c + a = \frac{13a}{9} > b$

It implies that a, b, c form a triangle with a as the greatest side.

Now, let us find the greatest angle A of $\Delta\ ABC$ by using the cosine formula.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{\frac{4a^2}{9} + \frac{16a^2}{81} - a^2}{\frac{4a}{3} \cdot \frac{4a}{9}}$$

$$= -\frac{29}{48} < 0$$

$\therefore$ The angle A is obtuse.

9 **(b)**

Required sum

$$= \sum_{n=0}^{\infty} \frac{(\log_e x)^n}{n\,!} = e^{\log_e x} = x$$

10 **(b)**

Sum of an infinite GP $= \frac{a}{1-r} = S$

$\Rightarrow a = S(1-r) \Rightarrow r = \frac{S-a}{S}$

11 **(b)**

We have,

$$\sum_{n=1}^{\infty} \frac{2\,n}{(2n+1)\,!} = \sum_{n=1}^{\infty} \frac{2n+1-1}{(2n+1)\,!}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{2\,n}{(2n+1)\,!} = \sum_{n=1}^{\infty} \left\{\frac{1}{(2n)\,!} - \frac{1}{(2n+1)\,!}\right\}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{2\,n}{(2n+1)} = \frac{1}{2\,!} - \frac{1}{3\,!} + \frac{1}{4\,!} - \frac{1}{5\,!} + \frac{1}{6\,!} - \frac{1}{7\,!} + \dots = e^{-1}$$

12 **(d)**

Let $S = 1 + 3x + 6x^2 + 10x^3 + \dots\infty \quad \dots(i)$

$xS = x + 3x^2 + 6x^3 + \dots\infty \quad \dots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$S(1-x) = 1 + 2x + 3x^2 + 4x^3 + \dots\infty \quad \dots(iii)$

$\Rightarrow x(1-x)S = x + 2x^2 + 3x^3 + \dots\infty \quad \dots(iv)$

Again, subtracting Eq. (iv) from Eq. (iii), we get

$S[(1-x) - x(1-x)] = (1 + x + x^2 + x^3 + \dots\infty)$

$$\Rightarrow S[(1-x)(1-x)] = \frac{1}{1-x}$$

$$\Rightarrow S = \frac{1}{(1-x)^3}$$

13 **(a)**

We have, $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \dots\infty$

$$\frac{1}{n!}({}^nC_o + {}^nC_2 + {}^nC_4 + \dots\infty) = \frac{2^{n-1}}{n!}$$

14 **(b)**

Let the first term and common difference of the A.P. be a and d respectively. Then,

Middle term $= 30 \Rightarrow 6th$ term $= 30 \Rightarrow a + 5d = 30$

Now,

$$S_{11} = \frac{11}{2}\{2a + 10d\} = 11 \times (a + 5d) = 11 \times 30 = 330$$

15 **(c)**

We have,

$(2.3)^x = (0.23)^y = 1000$

$\Rightarrow 2.3 = 10^{3/x}$ and $0.23 = 10^{3/y}$

$\Rightarrow 2.3 = 10^{3/x}$ and $2.3 = 10^{3/y+1}$

$\Rightarrow \log_{10} 2.3 = \frac{3}{x}$ and $\log_{10} 2.3 = \frac{3}{y} + 1$

$\Rightarrow \frac{3}{x} - \frac{3}{y} = 1 \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

17 **(b)**

We have,

$\log_6(x+3) - \log_6 x = 2$

$$\Rightarrow \log_6\left(\frac{x+3}{x}\right) = 2 \Rightarrow \frac{x+3}{x} = 6^2 \Rightarrow x + 3 = 36x \Rightarrow x = \frac{3}{35}$$

18 **(d)**

Let $S = 1 + 3 + 7 + 15 + \dots + T_n$

$$\Rightarrow \frac{S = 1 + 3 + 7 + \ldots + T_{n-1} + T_n}{0 = 1 + 2 + 4 + 8 + \ldots - T_n}$$

$\Rightarrow T_n = 1 + 2 + 4 + \ldots n$ terms

$$= \frac{(2^n - 1)}{2 - 1} = 2^n - 1$$

$$\therefore \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \ldots = \sum \frac{T_n}{2^n} = \sum \frac{2^n - 1}{2^n}$$

$$= \sum (1 - 2^{-n})$$

$$= n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1/2} = 2^{-n} + n - 1$$

19 **(a)**

Since, $T_7 = a + 6d = 40$ ...(i)

and $S_{13} = \frac{13}{2}[2a + 12d]$

$= 13[a + 6d]$

$= 13 \times 40 = 520$ [from Eq. (i)]

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	B	D	C	B	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	A	B	C	A	B	D	A	A