DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :3

Topic :-SEQUENCES AND SERIES

1 **(c)**

$\left(a+\frac{1}{p}-\frac{1}{q}\right)^2+\left(a+\frac{1}{q}-\frac{1}{r}\right)^2+\left(a+\frac{1}{r}-\frac{1}{s}\right)^2 \le 0$

$\Rightarrow \frac{1}{p}-\frac{1}{q}=\frac{1}{q}-\frac{1}{r}=\frac{1}{r}-\frac{1}{s}$

$\Rightarrow p,q,r,s$ are in HP.

2 **(d)**

$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\dots=\left(1-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\dots$

$=2\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\right)-1=\log_e\frac{4}{e}$

3 **(b)**

We have,

$x^{\log_x(x^2-4x+5)}=x-1$

$\Rightarrow x^2-4x+5=x-1 \Rightarrow x^2-5x+6=0 \Rightarrow x=2,3$

4 **(b)**

It is given that

$y-x, 2(y-a), (y-z)$ are in H.P.

$\Rightarrow \frac{1}{y-x},\frac{1}{2(y-a)},\frac{1}{y-z}$ are in A.P.

$\Rightarrow \frac{1}{2(y-a)}-\frac{1}{y-x}=\frac{1}{y-z}-\frac{1}{2(y-a)}$

$\Rightarrow \frac{2\,a-y-x}{y-x}=\frac{y+z-2a}{y-z}$

$\Rightarrow \frac{(x-a)+(y-a)}{(x-a)-(y-a)}=\frac{(y-a)+(z-a)}{(y-a)-(z-a)}$

$\Rightarrow \frac{x-a}{y-a}=\frac{y-a}{z-a}$

$x-a,\ y-a,\ z-a$ are in G.P.

5 **(b)**

The some of n terms of given series$=\frac{n(n+1)^2}{2}$ if n is even. Let n is odd $ie, n=2m+1$

Then, $S_{2m+1}=S_{2m}+(2m+1)$th term

$= \frac{(n-1)n^2}{2} + n\text{th term}$

$= \frac{(n-1)n^2}{2} + n^2 \quad [\because n \text{ is odd} = 2m+1]$

$= n^2\left[\frac{n-1+2}{2}\right] = \frac{(n+1)n^2}{2}$

6 **(c)**

LHS $= \frac{1(1-\lambda^{n+1})}{1-\lambda} = \frac{1-\lambda^{n+1}}{1-\lambda}$

And RHS $= (1+\lambda)(1+\lambda^2)(1+\lambda^4)$
$(1+\lambda^8)(1+\lambda^{16})$

$= \frac{(1-\lambda)(1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})}{(1-\lambda)}$

$= \frac{(1-\lambda^2)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})}{1-\lambda}$

$= \frac{(1-\lambda^{32})}{1-\lambda}$

$\therefore \frac{1-\lambda^{n+1}}{1-\lambda} = \frac{1-\lambda^{32}}{1-\lambda}$

$\Rightarrow 1-\lambda^{n+1} = 1-\lambda^{32}$

$\therefore n+1 = 32 \Rightarrow n = 31$

7 **(a)**

$\because (x+1)+(x+4)+(x+7)+\dots+(x+28) = 155$

Let n be the number of terms in the AP on LHS.

$\therefore x+28 = (x+1)+(n-1)3$

$\Rightarrow n = 10$

$\therefore \frac{10}{2}[(x+1)+(x+28)] = 155$

$\Rightarrow x = 1$

8 **(a)**

Let $'r'$ be the common ratio,

$\therefore \frac{\sum_{n=1}^{100} a_{2n}}{\sum_{n=1}^{100} a_{2n-1}} = \frac{a_2+a_4+a_6+\dots+a_{200}}{a_1+a_3+a_5+\dots+a_{199}}$

$= \frac{a_1(r+r^3+r^5+\dots+r^{199})}{a_1(1+r^2+r^4+\dots+r^{198})} = r$

$\Rightarrow \frac{\alpha}{\beta} = r$

9 **(a)**

Let $S = 2+7+14+23+34+\dots+T_n$ ...(i)

and $S = 2+7+14+23+34+\dots T_{n-1}+T_n$ ...(ii)

On subtracting Eqs. (i) from (ii),we get

$\therefore S-S=2+[5+7+9+11+\dots+T_n-T_{n-1}]-T_n$

$\Rightarrow T_n=2+\left[\frac{n-1}{2}\{2\times 5+(n-2)2\}\right]$

$\Rightarrow T_n=2+(n-1)(n+3)$

$\therefore T_{99}=2+98\times 102=9998$

10 **(b)**

Here, $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\leq 0$

$\Rightarrow(a^2p^2-2abp+b^2)+(b^2p^2-2bcp+c^2)+(c^2p^2-2cdp+d^2)\leq 0$

$\Rightarrow(ap-b)^2+(bp-c)^2+(cp-d)^2\leq 0$

(Since sum of squares is never less than zero).

⇒ Each of the square is zero.

$\therefore (ap-b)^2=(bp-c)^2=(cp-d)^2=0$

$\Rightarrow p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

$\therefore$ a,b,c,d are in GP.

11 **(c)**

Since, $\dfrac{\frac{2p}{3}+\frac{2p}{3}+\frac{2p}{3}+\underbrace{\frac{3q}{5}+\dots+\frac{3q}{5}}_{5\text{times}}+\underbrace{\frac{4r}{7}+\dots\frac{4r}{7}}_{7\text{times}}}{15}$

$\geq 15\sqrt{\left(\frac{2p}{3}\right)^3\left(\frac{3q}{5}\right)^5\left(\frac{4r}{7}\right)^7}$ $(\because \text{AM}\geq\text{GM})$

$\Rightarrow\ p^3q^5r^7\ \frac{2^3 3^5 4^7}{3^3 5^5 7^7}\leq 1$

$\Rightarrow\ p^3q^5r^7\leq\frac{5^5\ 7^7}{2^3\ 3^2\ 4^7}$

12 **(c)**

Let $S=1+10+10^2+\dots+10^{90}$

$=\frac{1\cdot(10^{91}-1)}{10-1}=\frac{(10^{13})^7-1}{10^{13}-1}\times\frac{10^{13}-1}{10-1}$

$=\left[(10^{13})^6+(10^{13})^5+(10^{13})^4+\dots+1\right]\times(10^{12}+10^{11}+\dots+)$

$\therefore$ It is the product of two integers and hence not prime.

13 **(d)**

Let $S=1+2x+3x^2+4x^3+\dots\infty$ ...(i)

$xS=x+2x^2+3x^3+\dots\infty$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$(1-x)S=1+x+x^2+x^3+\dots\infty$

$\Rightarrow S=\frac{1}{(1-x)}\left(\frac{1}{1-x}\right)=\frac{1}{(1-x)^2}$

14 **(c)**

We have,

$5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

$= 5^x - 7^{\frac{1}{x}}$, where $x = \sqrt{\log_5 7}$

$= 5^x - \left(5^{x^2}\right)^{\frac{1}{x}}\left[\because x = \sqrt{\log_5 7} \Rightarrow x^2 = \log_5 7 \Rightarrow 7 = 5^{x^2}\right]$

$= 5^x - 5^x = 0$

15 **(b)**

We have, $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \dots, \frac{1}{x_n}$ are in AP.

$\therefore \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = \dots = \frac{1}{x_n} - \frac{1}{x_{n-1}} = d$ (say)

$\therefore \frac{x_1 - x_2}{x_1 x_2} = \frac{x_2 - x_3}{x_2 x_3} = \dots = \frac{x_{m-1} - x_n}{x_{n-1} x_n} = d$

Now, $x_1 x_2 + x_2 x_3 + \dots + x_{n+1} x_n$

$= \frac{1}{d}[x_1 - x_2 + x_2 - x_3 + \dots + x_{n-1} - x_n]$

$= \frac{x_1 - x_n}{d}$

But $\frac{1}{x_n} = \frac{1}{x_1} + (n-1)d$

$\therefore \frac{x_1 - x_n}{x_1 x_n} = (n-1)d$

or $\frac{x_1 - x_n}{d} = (n-1)x_1 x_n$

$\therefore x_1 x_2 + x_2 x_3 + \dots + x_{n-1} x_n = (n-1) x_1 x_n$

16 **(d)**

Given a, b, c are in GP and $4a, 5b, 4c$ are in AP.

$\therefore b^2 = ac$ and $5b = \frac{4a + 4c}{2}$

$\Rightarrow b^2 = ac$ and $5b = 2a + 2c$

Now, $a + b + c = 70$ (given)

$\Rightarrow 2a + 2c + 2b = 140$

$\Rightarrow 5b + 2b = 140$

$\Rightarrow b = 20$

17 **(d)**

Since, p, q and r in HP.

$\Rightarrow q = \frac{2pr}{p+r} \Rightarrow \frac{q}{2} = \frac{pr}{p+r} = K$ (say)

$\Rightarrow q = 2K, pr = (p+r)K$

Also, p^2, q^2, r^2 are in AP.

$\therefore 2q^2 = p^2 + r^2 = (p+r)^2 - 2pr$

$\Rightarrow 8K^2 = (p+r)^2 - 2(p+r)K$

$\Rightarrow (p+r)^2 - 2(p+r)K - 8K^2 = 0$

$\Rightarrow p + r = 4K, -2K$

When $p + r = 4K$, then $pr = 4K^2$

$\therefore (p-r)^2 = (p+r)^2 - 4pr = 16K^2 - 16K^2 = 0$

$\Rightarrow p = r$

But this is not possible ($\because p \neq r$)

$\therefore p += -2K \Rightarrow pr = -2K \cdot K = -2K^2$

Now, $(p-r)^2 = (p+r)^2 - 4pr$

$= 4K^2 - 4(-2K^2) = 12\,K^2$

$\Rightarrow p - r = \pm\, 2\sqrt{3}K$

$\Rightarrow p = (-1 \pm \sqrt{3})K$

And $2r = -2K \mp \sqrt{3}K$

$\Rightarrow r = (-1 \mp \sqrt{3})K$

$\therefore p : q : r = (-1 \mp \sqrt{3})K : 2K : (-1 \mp \sqrt{3})K$

$= -1 \mp \sqrt{3} : 2 : -\ 1 \mp \sqrt{3}$

$= \ (-1 \mp \sqrt{3} : (-2) : (-1 \mp \sqrt{3})$

18 **(a)**

Given, $x = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots$

$\Rightarrow x = e^2$

$\Rightarrow x^{-1} = e^{-2}$

19 **(a)**

Since, $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty = \frac{\pi^4}{90}$

$$\Rightarrow \frac{\pi}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \ldots\infty\right) + \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \ldots\infty\right)$$

$$\Rightarrow \frac{\pi^4}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \ldots\infty\right) + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty\right)$$

$$\Rightarrow \frac{\pi^4}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \ldots\infty\right) + \frac{1}{16}\left(\frac{\pi^4}{90}\right)$$

$$\Rightarrow \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \ldots\infty$$

20 **(a)**

We have,

$(x+y) + (x^2 + xy + y^2) + (x^3 + x^2y + x\,y^2 + y^3) + \ldots\infty$

$$= \frac{x^2 - y^2}{x - y} + \frac{x^3 - y^3}{x - y} + \frac{x^4 - y^4}{x - y} + \ldots\text{to } \infty$$

$$= \frac{1}{x - y}\{(x^2 + x^3 + x^4 + \ldots) - (y^2 + y^3 + y^4 + \ldots)\}$$

$$= \frac{1}{x - y}\left\{\frac{x^2}{1 - x} - \frac{y^2}{1 - y}\right\} = \frac{x + y - xy}{1 - x - y + xy}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	B	B	B	C	A	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	D	C	B	D	D	A	A	A