DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :2

Topic :-SEQUENCES AND SERIES

1 **(c)**

Given, sum

$$= (x+2)^{n-1}\left\{1 + \left(\frac{x+1}{x+2}\right) + \left(\frac{x+1}{x+2}\right)^2 + \dots + \left(\frac{x+1}{x+2}\right)^{n-1}\right\}$$

$$= (x+2)^{n-1}\left\{\frac{1 - \left(\frac{x+1}{x+2}\right)^n}{1 - \left(\frac{x+1}{x+2}\right)}\right\}$$

$$= \frac{(x+2)^{n-1}\{(x+2)^n - (x+1)^n\} \cdot (x+2)}{(x+2)^n}$$

$$= (x+2)^n - (x+1)^n$$

2 **(a)**

Given $b^2 = ac,\ x = \frac{a+b}{2}$ and $y = \frac{b+c}{2}$

$$\therefore\ \frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$$

$$= \frac{2(2b+a+c)}{ab+b^2+bc+ac}$$

$$= \frac{2(2b+a+c)}{ab+2b^2+bc}$$

$$= \frac{2(2b+a+c)}{b(2b+a+c)}$$

$$= \frac{2}{b}$$

3 **(c)**

$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \dots$

$1 \times \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \dots$

$= \sqrt{2}(1 + 2 + 3 + 4 + \dots$upto 24 terms$)$

$$= \sqrt{2} \times \frac{24 \times 25}{2} = 300\sqrt{2} \quad \left[\because \Sigma n = \frac{n(n=1)}{2}\right]$$

4 **(c)**

Let $S = 1^3 + 2^3 + 3^3 + \ldots + 15^3$

$$= \sum_{n=1}^{15} n^3 = \left(\frac{15(15+1)}{2}\right)^2$$

$= 14400$

6 **(c)**

Since, $\Sigma n = \left(\frac{1}{5}\right)\Sigma n^2$

$$\Rightarrow \frac{n(n+1)}{2} = \frac{1}{5}\frac{n(n+1)(2n+1)}{6}$$

$\Rightarrow 2n + 1 = 15 \Rightarrow n = 7$

7 **(a)**

We know that, $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

Put $x = \frac{1}{2}$, we get

$$\frac{e^{1/2} + e^{-1/2}}{2} = 1 + \left(\frac{1}{2}\right)^2 \frac{1}{2!} + \left(\frac{1}{2}\right)^4 \frac{1}{4!} + \ldots$$

$$\Rightarrow \frac{e+1}{2\sqrt{e}} = 1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \ldots\infty$$

8 **(b)**

Let the three distinct real numbers in G.P. be a, ar, ar^2, where $r \neq \pm 1$

It is given that

$a^2 + a^2 r^2 + a^2 r^4 = S^2$

And,

$a + ar + ar^2 = \alpha S$

$$\therefore \frac{a^2(1 + r + r^2)^2}{a^2(1 + r^2 + r^4)} = \frac{\alpha^2 S^2}{S^2}$$

$$\Rightarrow \frac{(1 + r + r^2)^2}{(r^2 + r + 1)(r^2 - r + 1)} = \alpha^2$$

$$\Rightarrow \frac{r^2 + r + 1}{r^2 - r + 1} = \alpha^2$$

$\Rightarrow r^2(\alpha^2 - 1) - r(\alpha^2 + 1) + (\alpha^2 - 1) = 0$

$\Rightarrow (\alpha^2 + 1)^2 - 4(\alpha^2 - 1)^2 \geq 0 \qquad [\because r \text{ is real}]$

$\Rightarrow (3\alpha^2 - 1)(\alpha^2 - 3) \leq 0 \Rightarrow \frac{1}{3} \leq \alpha^2 \leq 3$

But, $\alpha^2 = 3$ for $r = 1$ and $\alpha^2 = \frac{1}{3}$ for $r = -1$

$$\therefore \frac{1}{3} < \alpha^2 < 3 \text{ } i.e. \text{ } \alpha^2 \in (1/3, 3)$$

9 **(c)**

Using, $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 1,\ r = \frac{1}{5},\ d = 3$

$\therefore S_\infty = \frac{1}{1-\frac{1}{5}} + \frac{3 \times \frac{1}{5}}{\left(1-\frac{1}{5}\right)^2}$

$= \frac{5}{4} + \frac{3}{5 \times \frac{16}{25}}$

$= \frac{5}{4} + \frac{15}{16} = \frac{35}{16}$

10 **(b)**

Since a,b,c are in HP.

$\therefore b = \frac{2\,ac}{a+c}$

$\Rightarrow \frac{b}{a} = \frac{2\,c}{a+c}$

$\Rightarrow \frac{b+a}{b-a} = \frac{3\,c+a}{c-a}$ [Applying componendo dividendo]

Again,

$b = \frac{2\,ac}{a+c}$

$\Rightarrow \frac{b}{c} = \frac{2\,a}{a+c}$

$\Rightarrow \frac{b+c}{b-c} = \frac{3\,a+c}{a-c}$

$\therefore \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{3\,c+a}{c-a} + \frac{3\,a+c}{a-c} = 2$

11 **(a)**

Here, $T_n = 1 + 2 + 2^2 + 2^3 + \dots + 2^n$

$= \frac{1(2^{n+1}-1)}{2-1} = 2^{n+1} - 1$

$\therefore \Sigma T_n = \Sigma\left(2^{n+1}-1\right) = \Sigma 2^{n+1} - \Sigma 1$

$= \left(2^2 + 2^3 + \dots + 2^{n+1}\right) - n$

$= 2\left(2 + 2^2 + \dots + 2^n\right) - n$

$= \frac{4(2^n-1)}{2-1} - n = 2^{n+2} - n - 4$

12 **(b)**

$\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} - \dots + \frac{3}{4} - \frac{3}{4}$

$$= 1 - \frac{1}{4} + \frac{1.3}{2.4.4} - \frac{1.3.5}{4.4.2.4.3} + \dots - \frac{3}{4}$$

$$= \left[1 + \frac{1}{1!}\left(-\frac{1}{4}\right) + \frac{1(1+2)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{1(1+2)(1+4)}{3!}\left(-\frac{1}{4}\right)^3 + \dots\right] - \frac{3}{4}$$

$$= \left(1 - \frac{1}{4}\right)^{-1/2} - \frac{3}{4} = \sqrt{\frac{2}{3}} - \frac{3}{4}$$

13 **(c)**

$$\therefore 1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \dots = e^{-\log 2}$$

$$= e^{\log 2^{-1}} = \frac{1}{2}$$

14 **(c)**

Let S_n and S' be the sums of n terms of two AP's and T_{11} and T'_{11} be the respective 11th term, then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}(2a' + (n-1)d']} = \frac{7n+1}{4n+27} \quad \text{(given)}$$

$$\Rightarrow \frac{a + \frac{(n-1)}{2}d}{a' + \frac{(n-1)}{2}d'} = \frac{7n+1}{4n+27}$$

Now put, $n = 21$, we get

$$\frac{a + 10d}{a' + 10d'} = \frac{T_{11}}{T'_{11}} = \frac{148}{111}$$

$$= \frac{4}{3}$$

15 **(a)**

$$2.\overline{357} = 2 + 0.357 + 0.000357 + \dots$$

$$\Rightarrow 2.\overline{357} = 2 + \frac{357}{10^3} + \frac{357}{10^6} + \dots$$

$$\Rightarrow 2.\overline{357} = 2 + \frac{\frac{357}{10^3}}{1 - \frac{1}{10^3}} = 2 + \frac{357}{999} = \frac{2355}{999}$$

16 **(c)**

Here, $T_n = \frac{1+2+3+\dots+n}{n!} = \frac{n(n+1)}{2(n)!}\left[\because \sum n = \frac{n(n+1)}{2}\right]$

$$= \frac{(n+1)}{2(n-1)!} = \frac{1}{2(n-2)!} + \frac{1}{(n-1)!}$$

$$T_1 = 0 + \frac{1}{1}$$

$$T_2 = \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{1!}$$

$$T_3 = \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{2!}$$

$T_4 = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3!}$

$\vdots \quad \vdots$

$\vdots \quad \vdots$

$\therefore S = \Sigma T_n$

$= \frac{1}{2}\left(1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\right) + \left(1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\right)$

$= \frac{1}{2}e + e = \frac{3}{2}e$

17 **(b)**

Let $S = 3 + 7 + 13 + 21 + \ldots + T_n$

$\Rightarrow T_n = n^2 + n + 1$

Let $T_r = \cot^{-1}(r^2 + r + 1)$

$= \tan^{-1}(r+1) - \tan^{-1} r$

Put $r = 1, 2, \ldots, n$

$T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 3 - \tan^{-1} 2$

$\ldots\ldots\ldots$

$\ldots\ldots\ldots$

$T_n = \tan^{-1}(n+1) - \tan^{-1} n$

On adding all these, we get

$T_1 + T_2 + \ldots + T_n = \tan^{-1}(n+1) - \tan^{-1} 1 = \tan^{-1}\left(\frac{n}{n+2}\right) = \cot^{-1}\left(\frac{n+2}{n}\right)$

19 **(a)**

Let $x^{18} = y^{21} = z^{28} = k$

Then,

$18 \log x = 21 \log y = 28 \log z = \log k$

$\Rightarrow \log_y x = \frac{21}{18}, \log_z y = \frac{28}{21}, \log_x z = \frac{18}{28}$

$\Rightarrow 3 \log_y x = \frac{7}{2}, 3 \log_z y = 4, 7 \log_x z = \frac{9}{2}$

$\Rightarrow 3, 3\log_y z, 3 \log_z y, 7 \log_x z$ are in A.P.

20 **(b)**

For $0 < x < \pi/2$, we have $0 < \sin^2 x < 1$

$\therefore y = \exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots\infty) \log_e 2]$

$\Rightarrow y = \exp\left[\left(\frac{\sin^2 x}{1 - \sin^2 x}\right) \log_e 2\right] = \exp[\tan^2 x \log_2 2]$

$\Rightarrow y = e^{\log_e^{2^{\tan^2 x}}} = 2^{\tan^2 x}$

Since y satisfies the equation $x^2 - 9x + 8 = 0$. Therefore,

$y^2 - 9y + 8 = 0 \Rightarrow (y-1)(y-8) = 0 \Rightarrow y = 1$ or, $y = 8$

Now,

$y = 1 \Rightarrow 2^{\tan^2 x} = 1 \Rightarrow 2^{\tan^2 x} = 2^0 \Rightarrow \tan x = 0 \Rightarrow x = 0$

But, $0 < x < \pi/2$. Therefore, $y \neq 1$.

Consequently, we have

$y = 8 \Rightarrow 2^{\tan^2 x} = 2^3 \Rightarrow \tan^2 x = 3 \Rightarrow \tan x = \sqrt{3} \Rightarrow x = \pi/3$

$$\therefore \frac{\sin x + \cos x}{\sin x - \cos x} = \frac{\tan x + 1}{\tan x - 1} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	C	D	C	A	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	C	A	C	B	B	A	B