DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :10

Topic :-SEQUENCES AND SERIES

1 **(a)**

Given, $S_\infty = \frac{4}{3}$ and $a = \frac{3}{4}$

Let r be the common ratio.

$\therefore \frac{a}{1-r} = \frac{4}{3}$

$\Rightarrow \frac{4}{3} - \frac{4}{3} r = \frac{3}{4}$

$\Rightarrow \frac{16-9}{12} = \frac{4}{3} r$

$\Rightarrow \frac{7}{12} = \frac{4}{3} r$

$\Rightarrow r = \frac{7}{16}$

2 **(d)**

We have,

$\log_{10}\{98 + \sqrt{(x-6)^2}\} = 2$

$\Rightarrow 98 + |x-6| = 10^2$

$\Rightarrow |x-6| = 2 \Rightarrow x - 6 = \pm\ 2 \Rightarrow x = 8,4$

3 **(a)**

We have,

$$a = \sum_{n=1}^{\infty} \frac{2\,n}{(2\,n-1)\,!}$$

$$\Rightarrow a = \sum_{n=1}^{\infty} \frac{2n-1+1}{(2n-1)\,!}$$

$$\Rightarrow a = \sum_{n=1}^{\infty} \left\{\frac{1}{(2n-2)\,!} + \frac{1}{(2n-1)\,!}\right\}$$

$$\Rightarrow a = \left(1 + \frac{1}{1\,!}\right) + \left(\frac{1}{3\,!} + \frac{1}{2\,!}\right) + \left(\frac{1}{5\,!} + \frac{1}{4\,!} + \dots\right) = e$$

and,

$$b=\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)\,!}$$

$$\Rightarrow b=\sum_{n=1}^{\infty}\frac{2\,n+1-1}{(2n+1)\,!}$$

$$\Rightarrow b=\sum_{n=1}^{\infty}\left\{\frac{1}{(2\,n)\,!}-\frac{1}{(2n+1)\,!}\right\}$$

$$\Rightarrow b=\left(\frac{1}{2\,!}-\frac{1}{3\,!}\right)+\left(\frac{1}{4\,!}-\frac{1}{5\,!}\right)+\left(\frac{1}{6\,!}-\frac{1}{7\,!}\right)+\ldots=e^{-1}$$

$\therefore ab=e\cdot e^{-1}=1$

4 **(a)**

Let

$$S_n=1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots\text{to } n \text{ terms}$$

$$\Rightarrow S_n=1+\frac{2^2-1}{2}+\frac{2^3-1}{2^2}+\frac{2^4-1}{2^3}+\frac{2^5-1}{2^4}+\ldots+\frac{2^n-1}{2^{n-1}}$$

$$\Rightarrow S_n=(2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{2^2}\right)+\left(2-\frac{1}{2^3}\right)+\left(2-\frac{1}{2^4}\right)+\ldots+\left(2-\frac{1}{2^{n-1}}\right)$$

$$\Rightarrow S_n=2n-\left(1+\frac{1}{2}+\frac{1}{2^2}+\ldots\frac{1}{2^{n-1}}\right)$$

$$\Rightarrow S_n=2n-\left(\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right)=2(n-1)+\frac{1}{2^{n-1}}$$

5 **(b)**

Since, $x=\sum_{n=0}^{\infty}\cos^{2n}\phi$

$=1+\cos^2\phi+\cos^4\phi+\ldots$

$=\frac{1}{1-\cos^2\phi}=\frac{1}{\sin^2\phi}$ $[\because |\cos x|<1]$

Similarly, $y=\frac{1}{1-\sin^2\phi}=\frac{1}{\cos^2\phi}$

and $z=\frac{1}{1-\sin^2\phi\cos^2\phi}$

$$=\frac{1}{1-\frac{1}{x}\cdot\frac{1}{y}}=\frac{xy}{xy-1}$$

$\Rightarrow xyz=xy+z$

6 **(d)**

Since, $a+23d=100$ ...(i)

$$\therefore\ S_{47}=\frac{47}{2}[2a+46d]=47[a+23d]$$

$=47\times 100=4700$ [from Eq. (i)]

587 **(b)**

Since, a,b,c,d,e,f are six A.M.'s. between 2 and 12

$\therefore a+b+c+d+e+f=\frac{6}{2}(a+f)=\frac{6}{2}(2+12)=42$

8 **(a)**

We have,

$x=\log_2 3$ and $y=\log_{1/2} 5$

$\Rightarrow x=\log_2 3$ and $y=\log_{2^{-1}} 5$

$\Rightarrow x=\log_2 3$ and $y=-\log_2 5$

$\Rightarrow x>0$ and $y<0 \Rightarrow x>y$

10 **(b)**

If $2^2<x<2^3$, then $2<\log_2 x<3$

$\therefore 2<\log_2 7<3 \qquad [\because 2^2<7<2^3]$

Let $\log_2 7$ be a rational number equal to $\frac{m}{n}$, where $m,n\in Z$, $n\neq 0$. Then,

$7=2^{m/n}\Rightarrow 7^n=2^m$

This is not possible as LHS is an odd natural number and RHS is an even natural number

11 **(d)**

We have,

$4^{\log_9 3}+9^{\log_2 4}=10^{\log_x 83}$

$\Rightarrow 4^{\log_{3^2} 2}+9^{\log_2 2^2}=10^{\log_x 83}$

$\Rightarrow 4^{1/2}+9^2=10^{\log_x 83}$

$\Rightarrow 83=10^{\log_x 83}\Rightarrow \log_{10} 83 = \log_x 83 \Rightarrow x=10$

12 **(c)**

Since, the given series $\log_a x, \log_b x, \log_c x$ be in HP.

$\Rightarrow \frac{\log x}{\log a}, \frac{\log x}{\log b}, \frac{\log x}{\log c}$ are in HP.

$\Rightarrow \frac{\log a}{\log x}, \frac{\log b}{\log x}, \frac{\log c}{\log x}$ are in AP.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in AP.

$\therefore a, b, c$ are in GP.

14 **(c)**

Since, $a+ar=a(1+r)=12$ ...(i)

and $ar^2+ar^3=ar^2(1+r)=48$ ...(ii)

From Eqs. (i) and (ii),

$r^2=4$

$\Rightarrow r=-2$

(Since, the series is alternately sign, so we take negative values).

On putting the value of r in Eq. (i), we get $a=-12$

15 **(a)**

We have,

$2\left\{\frac{m-n}{m+n}+\frac{1}{3}\left(\frac{m-n}{m+n}\right)^3+\frac{1}{5}\left(\frac{m-n}{m+n}\right)^5+\ldots\right\}$

$$= \log\left(\frac{1+\frac{m-n}{m+n}}{1-\frac{m-n}{m+n}}\right) = \log\left(\frac{m}{n}\right)$$

16 **(c)**

$$\sum_{k=1}^{n}(k^2+2k) = \sum_{k=1}^{n}k^2 + 2\sum_{k=1}^{n}k$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+7)}{6}$$

17 **(a)**

Let a be the first term and d be the common difference of the A.P. Then, it is given that

$x = \frac{p}{2}[2\,a + (p-1)\,d]$,

$y = \frac{q}{2}[2\,a + (q-1)\,d]$,

$z = \frac{r}{2}[2\,a + (r-1)\,d]$

$\therefore \frac{2\,x}{p} = 2\,a + (p-1)d$ ...(i)

$\frac{2\,y}{q} = 2\,a + (q-1)\,d$ ...(ii)

$\frac{2\,z}{r} = 2\,a + (r-1)\,d$ ...(iii)

Multiplying (i), (ii), and (iii) by $(q-r)$,$(r-p)$ and $(p-q)$ respectively and adding, we get

$$\frac{2\,x}{p}(q-r) + \frac{2\,y}{q}(r-p) + \frac{2\,z}{r}(p-q) = 0$$

$$\Rightarrow \frac{x}{p}(q-r) + \frac{y}{q}(r-p) + \frac{z}{r}(p-q) = 0$$

18 **(b)**

It is given that

$1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P.

$\Rightarrow (3^{1-x}+2)^{1/2}, 3, (4 \times 3^x - 1)$ are in G.P

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow 3 + 2.3^x = 12(3x)^2 - 3(3^x)$

$\Rightarrow 12(3^x)^2 - 5(3^x) - 3 = 0$

$\Rightarrow (4.3^x - 3)(3.3^x + 1) = 0$

$\Rightarrow 3^x = \frac{3}{4}$ $[\because 3.3^x + 1 \neq 0]$

$\Rightarrow x = \log_3\left(\frac{3}{4}\right) \Rightarrow x = \log_3 3 - \log_3 4 = 1 - \log_3 4$

19 **(c)**

Given that, AM =8, GM =5, if α, β are the roots of quadratic equation, then the required quadratic equation is

$x^2 - x(\alpha + \beta) + \alpha\beta = 0$ ...(i)

Here, AM $= \frac{\alpha + \beta}{2} = 8 \Rightarrow \alpha + \beta = 16$

And GM $= \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 25$

From Eq. (I)

$x^2 - 16x + 25 = 0$

20 **(a)**

Let $S_n = \frac{1}{\sqrt{2} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{8}} + \dots + \frac{1}{\sqrt{3n-1} + \sqrt{3n+2}}$

$$\frac{\sqrt{2} - \sqrt{5}}{-3} + \frac{\sqrt{5} - \sqrt{8}}{-3} + \dots + \frac{\sqrt{3n-1} - \sqrt{3n+2}}{-3}$$

$$= -\frac{1}{3}(\sqrt{2} - \sqrt{3n+2}) = \frac{1}{3}(\sqrt{3n+2} - \sqrt{2})$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	A	B	D	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	D	C	A	C	A	B	C	A