DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :1

Topic :-SEQUENCES AND SERIES

1 **(b)**

If p,q,r,s are in A.P., then in an A.P. or a G.P. or an H.P. $a_1,a_2,a_3,\ldots$, the terms a_p,a_q,a_r are in A.P., G.P. or H.P. respectively

2 **(c)**

$$T_n=\frac{\frac{n(n+1)}{2.2}}{1^3+2^3+3^3+\ldots+n^3}$$

$$=\frac{\frac{n(n+1)}{4}}{\left(\frac{n(n+1)}{2}\right)^2}=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\therefore T_n=\Sigma\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}$$

3 **(a)**

Since, a_1, a_2, $a_3,\ldots,a_n$ are in AP.

Then, $a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}=d$

Where d is the common difference of the give AP

Also, $a_n=a_1+(n-1)d$

Then, by rationalizing each term

$$\frac{1}{\sqrt{a_2}+\sqrt{a_1}}+\frac{1}{\sqrt{a_3}+\sqrt{a_2}}+\ldots+\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$=\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1}+\frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2}+\ldots+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}$$

$$=\frac{1}{d}\left(\sqrt{a_2}-\sqrt{a_1}+\sqrt{a_3}-\sqrt{a_2}+\ldots+\sqrt{a_n}-\sqrt{a_{n-1}}\right.$$

$$=\frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)\times\frac{\sqrt{a_n}+\sqrt{a_1}}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{1}{d}\left(\frac{a_n - a_1}{\sqrt{a_n} + \sqrt{a_1}}\right) = \frac{1}{d}\left(\frac{(n-1)d}{\sqrt{a_n} + \sqrt{a_1}}\right) = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}}$$

4 **(a)**

We have,

$$\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \ldots\text{ad.inf.}$$

$$= -\log_e\left(1 - \frac{1}{x^2}\right)$$

$$= -\log_e\left(1 - \frac{2}{y+1}\right) \quad \left[\because y = 2x^2 - 1 \therefore x^2 = \frac{y+1}{2}\right]$$

$$= -\log_e\left(\frac{y-1}{y+1}\right) = \log_e\left(\frac{y+1}{y-1}\right)$$

5 **(c)**

Let the number of sides of the polygon be n.Then, the sum of interior angles of the polygon

$$= (2n-4)\frac{\pi}{4} = (n-2)\pi$$

Since, the angles are in AP and $a = 120°, d = 5$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$\Rightarrow n = 9, 16$

Take $n = 16$

$T_{16} = a + 15d = 120° + 15(5°) = 195°$, which is impossible, an interior angle cannot be greater than 180°.

Hence, $n = 9$

6 **(d)**

We have,

$\log_x\left(4.x^{\log_5 x} + 5\right) = 2\log_5 x$

$\Rightarrow \log_x\left(4.x^{\log_5 x} + 5\right) = \log_5 x^2$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{\log_5 x^2}$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{2\log_5 x}$

$\Rightarrow 4y + 5 = y^2$, where $y = x^{\log_5 x}$

$\Rightarrow y^2 - 4y - 5 = 0$

$\Rightarrow y = 5, -1$

$\Rightarrow x^{\log_5 x} = 5 \quad [\because y \neq -1]$

$\Rightarrow \log_5 x = \log_x 5$

$\Rightarrow (\log_5 x)^2 = 1 \Rightarrow \log_5 x = \pm 1 \Rightarrow x = 5, 5^{-1}$

7 **(d)**

Since, $x = 1 + a + a^2 + \ldots\infty$

$\Rightarrow x = \frac{1}{1-a} \quad \Rightarrow \; a = \frac{x-1}{x}$

Similarly, $b = \frac{y-1}{y}$ and $c = \frac{z-1}{z}$

Since, a, b, c are in AP.

$\therefore b = \frac{a+c}{2}$

$\Rightarrow \frac{y-1}{y} = \frac{\frac{x-1}{x} + \frac{z-1}{z}}{2}$

$\Rightarrow 2xz(y-1) = y[z(x-1) + x(z-1)]$

$\Rightarrow 2xz = xy + yz$

8 **(a)**

We have,

$x^{(3/2)(\log_2 x - 3)} = 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = \log_x 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = -3 \log_x 2$

$\Rightarrow \frac{1}{2}(\log_2 x - 3) = -\frac{1}{\log_2 x}$

$\Rightarrow (\log_2 x)^2 - 3(\log_2 x) + 2 = 0$

$\Rightarrow (\log_2 x - 1)(\log_2 x - 2) = 0$

$\Rightarrow \log_2 x = 1, 2 \Rightarrow x = 2, 2^2$

11 **(b)**

Since, a, b, c are in AP.

$\Rightarrow 2b = a + c$, then straight line $ax + by + c = 0$ will pass through$(1, -2)$ because it satisfies condition $a - 2b + c = 0$ or $2b = a + c$.

12 **(a)**

We have,

$\frac{e^x}{1-x} = B_0 + B_1 x + B_2 x^2 + \ldots + B_n x^n + \ldots$

$\Rightarrow \sum_{r=0}^{\infty} \frac{x^r}{r\,!} = (B_0 + B_1 x + B_2 x^2 + \ldots + B_n x^n + \ldots)(1-x)$

On equating the coefficients of x^n on both sides, we get

$\frac{1}{n\,!} = B_n - B_{n-1}$

13 **(b)**

We have,

$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

Applying componendo and dividendo rule, we get

$$\frac{2a}{2bx}=\frac{2b}{2cx}=\frac{2c}{2dx}$$
$$\Rightarrow \frac{a}{b}=\frac{b}{c}=\frac{c}{d}$$
$\Rightarrow b^2=ac$ and $c^2=bd$

$\Rightarrow a, b, c$ and b, c, d are in GP, therefore a, b, c, d are in GP.

14 **(c)**

We have,
$$\sum_{r=1}^{n}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$$
$$\Rightarrow \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$$
Let $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{5^2}+-\text{to }\infty=x$
$$\Rightarrow\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)+\frac{1}{2^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\right)=x$$
$$\Rightarrow \frac{\pi^2}{8}+\frac{x}{4}=x\Rightarrow x=\frac{\pi^2}{6}$$
15 **(c)**

It will take 10yr for Jairam to pay off Rs 10000 in 10 yearly installments.

$\therefore$ He pays 10% annual interest on remaining amount.

$\therefore$ Money given in the first year
$$=1000+\frac{10000\times 10}{100}=1000+1000$$
$=$ Rs 2000

Money given in second year

$=1000+$ interest of $(10000-1000)$

$=1000+\frac{9000\times 10}{100}=100+900=$ Rs 1900

Similarly, money paid in third year = Rs 1800 etc.

So, money given by Jairam in 10 yr will be Rs 2000, Rs 1900, Rs 1800, Rs 1700 …

Which is in arithmetic progression, whose first term

$a=2000$ and $d=-100$

Total money given in 10 yr

$=\frac{10}{2}[2(2000)+(10-1)(-100)]=$ Rs 15500

Therefore, total money given by Jairam

$=5000+15500=$ Rs 20500

16 **(a)**

We have,

a,b,c are in A.P. ...(i)

$\Rightarrow\frac{a}{abc},\frac{b}{abc},\frac{c}{abc}$ are in A.P. $\Rightarrow\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ are in A.P. ...(ii)

From (i) and (ii), we obtain

$a+\frac{1}{bc}, b+\frac{1}{ca}, c+\frac{1}{ab}$ are in A.P.

17 **(d)**

We observe that the successive differences of the terms of the sequence 12,28,50,78,... are in A.P.

So, let its n^{th} term be

$t_n = an^2 + bn + c,$

Putting $n = 1,2,3$, we get

$t_1 = a + b + c \Rightarrow a + b + c = 12$

$t_2 = 4a + 2b + c \Rightarrow 4a + 2b + c = 28$

$t_3 = 9a + 3b + c \Rightarrow 9a + 3b + c = 50$

Solving these equations, we get

$a = 3, b = 7, c = 2$

$\therefore t_n = 3n^2 + 7n + 2$

Hence,

$$\frac{12}{2!}+\frac{28}{3!}+\frac{50}{4!}+\frac{78}{5!}+\ldots$$

$$=\sum_{n=1}^{\infty}\frac{3n^2+7n+2}{(n+1)!}$$

$$=\sum_{n=2}^{\infty}\frac{3(n-1)^2+7(n-1)+2}{n!}$$

$$=\sum_{n=2}^{\infty}\frac{3n^2+n-2}{n!}$$

$$=3\sum_{n=2}^{\infty}\frac{n^2}{n!}+\sum_{n=2}^{\infty}\frac{n}{n!}-2\sum_{n=2}^{\infty}\frac{1}{n!}$$

$$=2(2e-1)+(e-1)-2(e-2)=5e$$

18 **(b)**

We have,

$$\frac{x^2}{2}+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+\ldots$$

$$=\sum_{n=1}^{\infty}\frac{n}{n+1}\ x^{n+1}$$

$$=\sum_{n=1}^{\infty}\frac{n+1-1}{n+1}\ x^{n+1}$$

$$=\sum_{n=1}^{\infty}\left(1-\frac{1}{n+1}\right)x^{n+1}$$

$$=\sum_{n=1}^{\infty}x^{n+1}-(\overset{\infty}{n=1}\frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x - \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x + \log(1-x) = \frac{x}{1-x} + \log(1-x)$$

19 **(c)**

$$\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)^4 + \frac{1}{5}\left(\frac{1}{3}\right)^6 + \dots$$

$$= \frac{1}{3}\left[\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \dots\right]$$

$$= \frac{1}{3}\cdot\frac{1}{2}\log\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) \quad \left[\because \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right]$$

$$= \frac{1}{6}\log_e 2$$

20 **(a)**

Since, $T_m = \frac{1}{n} \Rightarrow a + (m-1)d$ ...(i)

and $T_n = \frac{1}{m} = a + (n-1)d$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = \frac{1}{mn}$ and $d = \frac{1}{mn}$

$$\therefore\ a - d = \frac{1}{mn} - \frac{1}{mn} = 0$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	A	A	C	D	D	A	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	C	C	A	D	B	C	A