

CLASS : XIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :1

Topic :-SEQUENCES AND SERIES

1 **(b)**

If p, q, r, s are in A.P., then in an A.P. or a G.P. or an H.P. a_1, a_2, a_3, \dots , the terms a_p, a_q, a_r are in A.P., G.P. or H.P. respectively

2 **(c)**

$$\begin{aligned} T_n &= \frac{\frac{n(n+1)}{2}}{\frac{1^3 + 2^3 + 3^3 + \dots + n^3}{2}} \\ &= \frac{\frac{n(n+1)}{4}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \\ \therefore T_n &= \sum \left(\frac{1}{n} - \frac{1}{n+1} \right) \\ &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right) \\ &= 1 - \frac{1}{n+1} = \frac{n}{n+1} \end{aligned}$$

3 **(a)**

Since, $a_1, a_2, a_3, \dots, a_n$ are in AP.

Then, $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = d$

Where d is the common difference of the give AP

Also, $a_n = a_1 + (n-1)d$

Then, by rationalizing each term

$$\begin{aligned} &\frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + \dots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \\ &= \frac{1}{d} (\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \dots + \sqrt{a_n} - \sqrt{a_{n-1}}) \\ &= \frac{1}{d} (\sqrt{a_n} - \sqrt{a_1}) \times \frac{\sqrt{a_n} + \sqrt{a_1}}{\sqrt{a_n} + \sqrt{a_1}} \end{aligned}$$

$$= \frac{1}{d} \left(\frac{a_n - a_1}{\sqrt{a_n} + \sqrt{a_1}} \right) = \frac{1}{d} \left(\frac{(n-1)d}{\sqrt{a_n} + \sqrt{a_1}} \right) = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}}$$

4 (a)

We have,

$$\begin{aligned} & \frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \dots \text{ad.inf.} \\ & = -\log_e \left(1 - \frac{1}{x^2} \right) \\ & = -\log_e \left(1 - \frac{2}{y+1} \right) \quad \left[\because y = 2x^2 - 1 \therefore x^2 = \frac{y+1}{2} \right] \\ & = -\log_e \left(\frac{y-1}{y+1} \right) = \log_e \left(\frac{y+1}{y-1} \right) \end{aligned}$$

5 (c)

Let the number of sides of the polygon be n . Then, the sum of interior angles of the polygon

$$= (2n-4) \frac{\pi}{4} = (n-2)\pi$$

Since, the angles are in AP and $a = 120^\circ$, $d = 5$

$$\text{Therefore, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$$

$$\Rightarrow n^2 - 25n + 144 = 0$$

$$\Rightarrow (n-9)(n-16) = 0$$

$$\Rightarrow n = 9, 16$$

Take $n = 16$

$T_{16} = a + 15d = 120^\circ + 15(5^\circ) = 195^\circ$, which is impossible, an interior angle cannot be greater than 180° .

Hence, $n = 9$

6 (d)

We have,

$$\log_x(4x^{\log_5 x} + 5) = 2 \log_5 x$$

$$\Rightarrow \log_x(4x^{\log_5 x} + 5) = \log_5 x^2$$

$$\Rightarrow 4x^{\log_5 x} + 5 = x^{\log_5 x^2}$$

$$\Rightarrow 4x^{\log_5 x} + 5 = x^{2 \log_5 x}$$

$$\Rightarrow 4y + 5 = y^2, \text{ where } y = x^{\log_5 x}$$

$$\Rightarrow y^2 - 4y - 5 = 0$$

$$\Rightarrow y = 5, -1$$

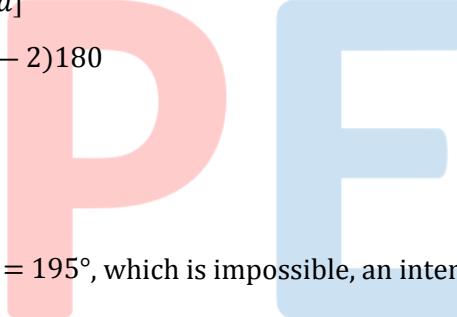
$$\Rightarrow x^{\log_5 x} = 5 \quad [\because y \neq -1]$$

$$\Rightarrow \log_5 x = \log_5 5$$

$$\Rightarrow (\log_5 x)^2 = 1 \Rightarrow \log_5 x = \pm 1 \Rightarrow x = 5, 5^{-1}$$

7 (d)

Since, $x = 1 + a + a^2 + \dots \infty$



$$\Rightarrow x = \frac{1}{1-a} \Rightarrow a = \frac{x-1}{x}$$

$$\text{Similarly, } b = \frac{y-1}{y} \text{ and } c = \frac{z-1}{z}$$

Since, a, b, c are in AP.

$$\therefore b = \frac{a+c}{2}$$

$$\Rightarrow \frac{y-1}{y} = \frac{\frac{x-1}{x} + \frac{z-1}{z}}{2}$$

$$\Rightarrow 2xz(y-1) = y[z(x-1) + x(z-1)]$$

$$\Rightarrow 2xz = xy + yz$$

8 **(a)**

We have,

$$x^{(3/2)(\log_2 x - 3)} = 2^{-3}$$

$$\Rightarrow \frac{3}{2}(\log_2 x - 3) = \log_x 2^{-3}$$

$$\Rightarrow \frac{3}{2}(\log_2 x - 3) = -3 \log_x 2$$

$$\Rightarrow \frac{1}{2}(\log_2 x - 3) = -\frac{1}{\log_2 x}$$

$$\Rightarrow (\log_2 x)^2 - 3(\log_2 x) + 2 = 0$$

$$\Rightarrow (\log_2 x - 1)(\log_2 x - 2) = 0$$

$$\Rightarrow \log_2 x = 1, 2 \Rightarrow x = 2, 2^2$$

11 **(b)**

Since, a, b, c are in AP.

$\Rightarrow 2b = a + c$, then straight line $ax + by + c = 0$ will pass through $(1, -2)$ because it satisfies condition $a - 2b + c = 0$ or $2b = a + c$.

12 **(a)**

We have,

$$\frac{e^x}{1-x} = B_0 + B_1x + B_2x^2 + \dots + B_nx^n + \dots$$

$$\Rightarrow \sum_{r=0}^{\infty} \frac{x^r}{r!} = (B_0 + B_1x + B_2x^2 + \dots + B_nx^n + \dots)(1-x)$$

On equating the coefficients of x^n on both sides, we get

$$\frac{1}{n!} = B_n - B_{n-1}$$

13 **(b)**

We have,

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

Applying componendo and dividendo rule, we get



$$\frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx}$$

$$\Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

$$\Rightarrow b^2 = ac \text{ and } c^2 = bd$$

$\Rightarrow a, b, c$ and b, c, d are in GP, therefore a, b, c, d are in GP.

14 (c)

We have,

$$\sum_{r=1}^n \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$$

$$\Rightarrow \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

$$\text{Let } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = x$$

$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \right) + \frac{1}{2^2} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right) = x$$

$$\Rightarrow \frac{\pi^2}{8} + \frac{x}{4} = x \Rightarrow x = \frac{\pi^2}{6}$$

15 (c)

It will take 10yr for Jairam to pay off Rs 10000 in 10 yearly installments.

\therefore He pays 10% annual interest on remaining amount.

\therefore Money given in the first year

$$= 1000 + \frac{10000 \times 10}{100} = 1000 + 1000$$

$$= \text{Rs } 2000$$

Money given in second year

$$= 1000 + \text{interest of } (10000 - 1000)$$

$$= 1000 + \frac{9000 \times 10}{100} = 100 + 900 = \text{Rs } 1900$$

Similarly, money paid in third year = Rs 1800 etc.

So, money given by Jairam in 10 yr will be Rs 2000, Rs 1900, Rs 1800, Rs 1700 ...

Which is in arithmetic progression, whose first term

$$a = 2000 \text{ and } d = -100$$

Total money given in 10 yr

$$= \frac{10}{2} [2(2000) + (10 - 1)(-100)] = \text{Rs } 15500$$

Therefore, total money given by Jairam

$$= 5000 + 15500 = \text{Rs } 20500$$

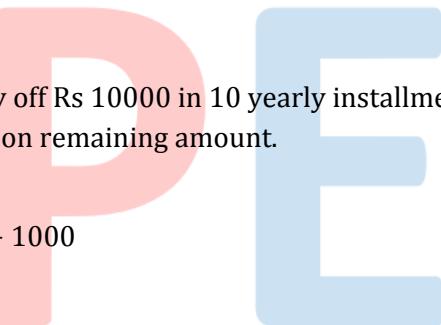
16 (a)

We have,

a, b, c are in A.P. ... (i)

$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are in A.P. $\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P. ... (ii)

From (i) and (ii), we obtain



$a + \frac{1}{bc}, b + \frac{1}{ca}, c + \frac{1}{ab}$ are in A.P.

17 (d)

We observe that the successive differences of the terms of the sequence 12,28,50,78,... are in A.P.

So, let its n^{th} term be

$$t_n = an^2 + bn + c,$$

Putting $n = 1, 2, 3$, we get

$$t_1 = a + b + c \Rightarrow a + b + c = 12$$

$$t_2 = 4a + 2b + c \Rightarrow 4a + 2b + c = 28$$

$$t_3 = 9a + 3b + c \Rightarrow 9a + 3b + c = 50$$

Solving these equations, we get

$$a = 3, b = 7, c = 2$$

$$\therefore t_n = 3n^2 + 7n + 2$$

Hence,

$$\frac{12}{2!} + \frac{28}{3!} + \frac{50}{4!} + \frac{78}{5!} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{3n^2 + 7n + 2}{(n+1)!}$$

$$= \sum_{n=2}^{\infty} \frac{3(n-1)^2 + 7(n-1) + 2}{n!}$$

$$= \sum_{n=2}^{\infty} \frac{3n^2 + n - 2}{n!}$$

$$= 3 \sum_{n=2}^{\infty} \frac{n^2}{n!} + \sum_{n=2}^{\infty} \frac{n}{n!} - 2 \sum_{n=2}^{\infty} \frac{1}{n!}$$

$$= 2(2e - 1) + (e - 1) - 2(e - 2) = 5e$$

18 (b)

We have,

$$\frac{x^2}{2} + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \dots$$

$$= \sum_{n=1}^{\infty} \frac{n}{n+1} x^{n+1}$$

$$= \sum_{n=1}^{\infty} \frac{n+1-1}{n+1} x^{n+1}$$

$$= \sum_{n=1}^{\infty} \left(1 - \frac{1}{n+1}\right) x^{n+1}$$

$$= \sum_{n=1}^{\infty} x^{n+1} - \left(\sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1}\right)$$



$$= \frac{x^2}{1-x} + x - \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x + \log(1-x) = \frac{x}{1-x} + \log(1-x)$$

19 **(c)**

$$\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)^4 + \frac{1}{5}\left(\frac{1}{3}\right)^6 + \dots$$

$$= \frac{1}{3} \left[\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \dots \right]$$

$$= \frac{1}{3} \cdot \frac{1}{2} \log \left(\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} \right) \quad \left[\because \frac{1}{2} \log \left(\frac{1+x}{1-x} \right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right]$$

$$= \frac{1}{6} \log_e 2$$

20 **(a)**

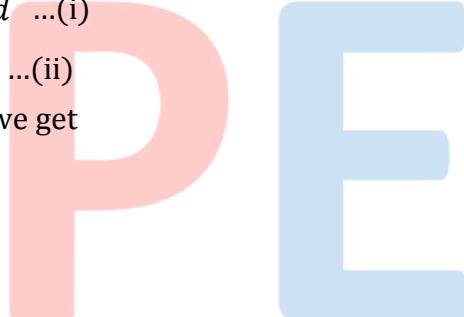
$$\text{Since, } T_m = \frac{1}{n} \Rightarrow a + (m-1)d \quad \dots(i)$$

$$\text{and } T_n = \frac{1}{m} = a + (n-1)d \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{mn} \text{ and } d = \frac{1}{mn}$$

$$\therefore a - d = \frac{1}{mn} - \frac{1}{mn} = 0$$



ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	A	A	C	D	D	A	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	C	C	A	D	B	C	A

P E