DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :9

Topic :-RELATIONS AND FUNCTIONS

1 **(c)**

The period of the function in option (a) is 2. The period of the function in option (b) is 24.
The period of the function in option (c) is 2π.

2 **(a)**

We have,

$f(x) = \sqrt{3} \sin x + \cos x + 4$

$\Rightarrow f(x) = 2(\sin x \cos \pi/6 + \cos x \sin \pi/6) + 4$

$\Rightarrow f(x) = 2 \sin(x + \pi/6) + 4$

Clearly, $f(x)$ will be a bijection, if $\sin(x + \pi/6)$ is a bijection

Now,

$\sin(x + \pi/6)$ is a bijection

$\Rightarrow -\pi/2 \leq x + \pi/6 \leq \pi/2$

$\Rightarrow -2\pi/3 \leq x \leq \pi/3$

$\Rightarrow x \in [-2\pi/3, \pi/3]$

For $x \in [-2/3\pi, \pi/3]$, we have

$-1 \leq \sin(x + \pi/6) \leq 1$

$\Rightarrow -2 \leq 2\sin(x + \pi/6) \leq 2$

$\Rightarrow -2 + 4 \leq 2\sin(x + \pi/6) + 4 \leq 2 + 4$

$\Rightarrow 2 \leq f(x) \leq 6$

$\Rightarrow$Range of $f(x) = [2, 6]$

Hence, $A = [-2\pi/3, \pi/3]$ and $B = [2, 6]$

3 **(c)**

We have,

$f(x) = 2x + 3$ and $g(x) = x^2 + 7$

$\therefore g(f(x)) = g(2x + 3) = (2x + 3)^2 + 7$

Now,

$g(f(x)) = 8$

$\Rightarrow (2x + 3)^2 + 7 = 8$

$\Rightarrow (2x + 3)^2 = 1$

$\Rightarrow 2x + 3 = \pm 1 \Rightarrow 2x = -4, -2 \Rightarrow x = -1, -2$

4 **(c)**

We have,

$$f(x) = \sin^{-1}\left(\frac{x-3}{2}\right) - \log(4-x) \ = g(x) + h(x)$$

where $g(x) = \sin^{-1}\left(\frac{x-3}{2}\right)$ and $h(x) = -\log(4-x)$

now, $g(x)$ is defined for

$$-1 \leq \frac{x-3}{2} \leq 1 \Rightarrow -2 \leq x-3 \leq 2 \Rightarrow 1 \leq x \leq 5$$

and, $h(x)$ is defined for $4 - x > 0 \Rightarrow x < 4$

So, domain of $f(x) = [1, 5] \cap [-\infty, 4) = [1, 4)$

5 **(a)**

Let $y = f(x) = \frac{1-x}{1+x}$ $\qquad [\because x \neq -1]$

$$\Rightarrow \quad x = \frac{1-y}{1+y}$$

$$\therefore \quad f^{-1}(x) = \frac{1-x}{1+x} = f(x)$$

6 **(b)**

Since, $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ $\qquad$ ...(i)

Replacing x by $\frac{x+59}{x-1}$ in Eq. (i), we get

$\therefore \quad 3\left(\frac{x+59}{x-1}\right) + 2f(x) = \frac{40x+560}{x-1}$ $\qquad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$f(x) = \frac{6x^2 - 4x - 242}{x-1}$$

$$\therefore \quad f(7) = \frac{6 \times 49 - 28 - 242}{6} = 4$$

7 **(c)**

$$\left[\frac{2}{3} + \frac{r}{99}\right] = \begin{cases} 0, & r < 33 \\ 1, & r \geq 33 \end{cases}$$

$$\therefore \quad \sum_{r=0}^{98}\left[\frac{2}{3} + \frac{r}{99}\right] = \sum_{r=0}^{32}\left[\frac{2}{3} + \frac{r}{99}\right] + \sum_{r=33}^{98}\left[\frac{2}{3} + \frac{r}{99}\right]$$

$$= 0 + 66 = 66$$

8 **(b)**

We have, Domain $(f) = [0, 1]$

$\therefore f(3x^2)$ is defined, if

$0 \leq 3x^2 \leq 1$

$$\Rightarrow 0 \leq x^2 \leq \frac{1}{3} \Rightarrow |x| \leq \frac{1}{\sqrt{3}} \Rightarrow x \in [-1/\sqrt{3}, 1/\sqrt{3}]$$

9 **(d)**

$$\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$$

Since, $\quad -2 \leq 2\sin\left(x-\frac{\pi}{3}\right) \leq 2$

$\Rightarrow \quad -1 \leq 1 + 2\sin\left(x-\frac{\pi}{3}\right) \leq 3$

$\therefore$ Range of $S = [-1, 3]$

10 **(b)**

Given,

$f(x) = e^x$ and $g(x) = \log_e x$

Now, $f\{g(x)\} = e^{\log_e x} = x$

And $g\{f(x)\} = \log_e e^x = x$

$\therefore f\{g(x)\} = g\{f(x)\}$

11 **(a)**

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i)$7 - x \geq 0$ (ii)$x - 3 \geq 0$ (iii)$7 - x \geq x - 3$

Now,

$$\left.\begin{aligned} 7 - x \geq 0 &\Rightarrow x \leq 7 \\ x - 3 \geq 0 &\Rightarrow x \geq 3 \\ 7 - x \geq x - 3 &\Rightarrow x \leq 5 \end{aligned}\right\} \Rightarrow 3 \leq x \leq 5$$

Hence, the required domain is $\{3, 4, 5\}$

Now,

$f(3) = {}^{7-3}P_0, f(4) = {}^{3}P_1 = 3$ and $f(5) = {}^{2}P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$

12 **(c)**

We have,

$f(x) = \log_{1.7}\left\{\frac{2-\varphi'(x)}{x+1}\right\}$, where $\varphi(x) = \frac{x^3}{3} - \frac{3}{2}x^2 - 2x + \frac{3}{2}$

For $f(x)$ to be defined, we must have

$\frac{2-\varphi'(x)}{x+1} > 0, x \neq -1$

$\Rightarrow \frac{2-(x^2-3x-2)}{3x+1} > 0, x \neq -1$

$\Rightarrow \frac{x^2-3x-4}{x+1} < 0, x \neq -1$

$\Rightarrow \frac{(x-4)(x+1)}{x+1} < 0, x \neq -1$

$\Rightarrow x - 4 < 0, x \neq -1$

$\Rightarrow x < 4, x \neq -1$

$\Rightarrow x \in (-\infty, 4), x \neq -1 \Rightarrow x \in (-\infty, -1) \cup (-1, 4)$

13 **(a)**

$f(x)$ is defined, if

$-1 \leq \frac{4}{3 + 2\cos x} \leq 1$

$\Rightarrow \frac{4}{3 + 2\cos x} \leq 1 \qquad [\because 3 + 2\cos x > 0]$

$\Rightarrow 4 \leq 3 + 2\cos x$

$\Rightarrow \cos x \geq \frac{1}{2} \Rightarrow 2n\pi - \frac{\pi}{6} \leq x \leq \frac{\pi}{6}, n \in Z$

14 **(c)**

The period of the function in (a) is 2. The period of the function in (b) is 24. The period of the function in (c) is 2π

15 **(a)**

$R = \{(a, b): 1 + ab > 0\}$

It is clear that the given relation on S is reflexive, symmetric but not transitive.

17 **(a)**

We have,

$f(x) = \max\{(1-x), 2, (1+x)\}$

For $x \leq -1$, we find that

$1 - x \geq 2$, and $1 - x \geq 1 + x$

$\therefore \text{Max}\{(1-x), 2, (1+x)\} = 1 - x$

For $-1 < x < 1$, we find that

$0 < 1 - x < 2$, and $0 < 1 + x < 2$

$\therefore \text{Max}\{(1-x), (1+x)\} = 2$

For $x \geq 1$, we observe that

$1 + x \geq 2, 1 + x > 1 - x$

$\therefore \text{Max}\{(1-x), 2, (1+x)\} = 1 + x$

Hence, $f(x) = \begin{cases} 1 - x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1 + x, & x \geq 1 \end{cases}$

<u>**NOTE**</u> *Students are advised to solve this problem by drawing the graphs of* $y = 1 - x, y = 2$ and $y = 1 + x$

18 **(d)**

Period of $\sin\frac{\theta}{3} = 6\pi$

And period of $\cos\frac{\theta}{2} = 4\pi$

$\therefore$ Period of $f(x) = \text{LCM}(6\pi, 4\pi) = 12\pi$

19 **(b)**

To make $f(x)$ an odd function in the interval $[-1,1]$, we re-define $f(x)$ as follows:

$f(x) = \begin{cases} f(x), & 0 \leq x \leq 1 \\ -f(-x), & -1 \leq x < 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 + x + \sin x - \cos x + \log(1 + |x|), & 0 \leq x \leq 1 \\ -(x^2 - x - \sin x - \cos x + \log(1 + |x|), & -1 \leq x < 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 + x + \sin x - \cos x + \log(1 + |x|), & 0 \leq x \leq 1 \\ -x^2 + x + \sin x + \cos x - \log(1 + |x|), & -1 \leq x < 0 \end{cases}$

Thus, the odd extension of $f(x)$ to the interval $[-1, 1]$ is

$-x^2 + x + \sin x + \cos x - \log(1 + |x|)$

20 **(b)**

We have,

$g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$

Now,

$f(g(x)) = 3 + 2\sqrt{x} + x$

$\Rightarrow f(g\ (x)) = 2 + (1 + \sqrt{x})^2$

$\Rightarrow f(g\ (x)) = 2 + \{g(x)\}^2$

$\Rightarrow f(x) = 2 + x^2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	C	A	B	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	A	C	A	D	A	D	B	B