

Topic :-RELATIONS AND FUNCTIONS

1 (c)

Given, $f(x) = x^3 - 1$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$$\Rightarrow x_1^3 - 1 = x_2^3 - 1$$

$$\Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow x_1 = x_2$$

$\therefore f(x)$ is one-one. Also, it is onto as range of $f = R$
Hence, it is a bijection.

2 (d)

Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f(g(\frac{8}{5})) = f(\frac{8}{5}) = [\frac{8}{5}] = 1$

And $g(f(-\frac{8}{5})) = g([\frac{-8}{5}]) = g(-2) = 2$

$$\therefore f(g(\frac{8}{5})) - g(f(-\frac{8}{5})) = 1 - 2 = -1$$

3 (a)

$$\therefore f(x) = \frac{\cos^{-1}x}{[x]}$$

For $f(x)$ to be defined $-1 \leq x \leq 1$ and $[x] \neq 0 \Rightarrow x \notin [0, 1)$

\therefore Domain of $f(x)$ is $[-1, 0) \cup \{1\}$.

4 (c)

Let $f(x) = g(x) + h(x) + u(x)$, where

$$g(x) = \frac{1}{x}, h(x) = 2^{\sin^{-1}x} \text{ and } u(x) = \frac{1}{\sqrt{x-2}}$$

The domain of $g(x)$ is the set of all real numbers other than zero i.e. $R - \{0\}$

The domain of $h(x)$ is the set $[-1, 1]$ and the domain of $u(x)$ is the set of all reals greater than 2, i.e., $(2, \infty)$

Therefore, domain of $f(x) = R - \{0\} \cap [-1, 1] \cap (2, \infty) = \phi$

5 (b)

Given, $2f(x) + f(1-x) = x^2$... (i)

Replacing x by $(1-x)$, we get

$$2f(1-x) + f(x) = (1-x)^2$$

$$\Rightarrow 2f(1-x) + f(x) = 1 + x^2 - 2x \quad \dots(ii)$$

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$3f(x) = x^2 + 2x - 1 \Rightarrow f(x) = \frac{x^2 - 2x - 1}{3}$$

6 (d)

$$f(x) = a + bx$$

$$\begin{aligned} \therefore f\{f(x)\} &= a + b(a + bx) = a(1 + b) + b^2x \\ \Rightarrow f[f\{f(x)\}] &= f\{a(1 + b) + b^2x\} = a(1 + b + b^2) + b^3x \\ \therefore f^r(x) &= a(1 + b + b^2 + \dots + b^{r-1}) + b^r x \\ &= a\left(\frac{b^r - 1}{b - 1}\right) + b^r x \end{aligned}$$

7 (b)

We have,

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{2x}{-2}$$

$$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$$

$$\therefore f(2x) = \frac{2x-1}{2x+1} = \frac{2\left(\frac{f(x)+1}{1-f(x)}\right) - 1}{2\left(\frac{f(x)+1}{1-f(x)}\right) + 1} = \frac{3f(x)+1}{f(x)+3}$$

8 (a)

Since, $f(-x) = -f(x)$ and $f(x+2) = f(x)$

$$\therefore f(x) = f(0) \text{ and } f(-2) = f(-2+2) = f(0)$$

Now, $f(0) = f(-2) = -f(2) = -f(0)$

$$\Rightarrow 2f(0) = 0 \Rightarrow f(0) = 0$$

$$\therefore f(4) = f(2) = f(0) = 0$$

9 (c)

We observe that $\frac{1}{x^2 - 36}$ is not defined for $x = \pm 6$

Also, $\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$ is a real number, if

$$0 < \frac{x-1}{x+5} \leq 1$$

$$\Rightarrow 0 < \frac{x-1}{x+5} \text{ and } \frac{x-1}{x+5} \leq 1$$

$$\Rightarrow (x-1)(x+5) > 0 \text{ and } 1 - \frac{6}{x+5} \leq 1$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{ and } -\frac{6}{x+5} \leq 0$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{ and } x+5 > 0$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{ and } x > -5$$

Hence, domain of $f(x) = (1, \infty) - \{6\}$

10 (b)

Given, $f(x) = \log_2(\log_3(\log_4 x))$

We know, $\log_a x$ is defined, if $x > 0$

For $f(x)$ to be defined.

$\log_3 \log_4 x > 0, \log_4 x > 0$ and $x > 0$

$\Rightarrow \log_4 x > 3^0 = 1, x > 4^0 = 1$ and $x > 0$

$\Rightarrow x > 4, x > 1$ and $x > 0$

$\Rightarrow x > 4$

11 (c)

We have,

$$f(x) = \begin{cases} -3x + 9, & \text{if } x < 2 \\ x - 3, & \text{if } 2 \leq x < 3 \\ x - 1, & \text{if } 3 \leq x < 4 \\ 3x - 9, & \text{if } x \geq 4 \end{cases}$$

$$\therefore g(x) = f(x+1) = \begin{cases} -3x + 6, & \text{if } x < 1 \\ x - 2, & \text{if } 1 \leq x < 2 \\ x, & \text{if } 2 \leq x < 3 \\ 3x - 6, & \text{if } x \geq 3 \end{cases}$$

Clearly, $g(x)$ is neither even nor odd. Also, $g(x)$ is not a periodic function

12 (b)

We have,

$f: [2, \infty) \rightarrow B$ such that $f(x) = x^2 - 4x + 5$

Since f is a bijection. Therefore, $B = \text{Range of } f$

Now,

$f(x) = x^2 - 4x + 5 = 5 = (x - 2)^2 + 1$ for all $x \in [2, \infty)$

$\Rightarrow f(x) \geq 1$ for all $x \in [2, \infty) \Rightarrow \text{Range of } f = [1, \infty)$

Hence, $B = [1, \infty)$

13 (d)

Given, $R = \{(x, y): 4x + 3y = 20\}$.

Since, R is a relation on N , therefore x, y are the elements of N . But in options (a) and (b) elements are not natural numbers and option (c) does not satisfy the given relation $4x + 3y = 20$.

14 (b)

Since the function $f: R \rightarrow R$ given by $f(x) = x^3 + 5$ is a bijection. Therefore, f^{-1} exists

Let $f(x) = y$. Then,

$x^3 + 5 = y$

$\Rightarrow x = (y - 5)^{1/3}$ [$\because f(x) = y \Leftrightarrow x = f^{-1}(y)$]

Hence, $f^{-1}(x) = (x - 5)^{1/3}$

15 (a)

We have,

$f(x) = x, g(x) = |x|$ for all $x \in R$

Now,

$[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

$\Rightarrow \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0$

$$\Rightarrow \phi(x) = f(x) \text{ and } \phi(x) = g(x)$$

$$\Rightarrow f(x) = g(x) = \phi(x)$$

But, $f(x) = g(x) = x$, for all $x \geq 0$ [$\because |x| = x$ for all $x \geq 0$]

$$\therefore \phi(x) = x \text{ for all } x \in [0, \infty)$$

16 **(b)**

Since $f(x)$ is defined for $x \in [0, 1]$. Therefore, $f(2x + 3)$ exists if

$$0 \leq 2x + 3 \leq 1 \Rightarrow -\frac{3}{2} \leq x \leq -1 \Rightarrow x \in [-3/2, -1]$$

18 **(a)**

$$\begin{aligned} f \circ g(-1) &= f\{g(-1)\} \\ &= f(-7) = 5 - 49 = -44 \end{aligned}$$

19 **(a)**

We have,

$$f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}} \text{ for all } x \in R$$

Clearly, $f(-x) = f(x)$ for all $x \in R$

So, f is a many-one function

$$\text{Also, } e^{x^2} > e^{-x^2} > 0$$

So, $f(x)$ attains only positive values

Consequently, range of $f \neq R$

Hence, f is many-one into function

20 **(c)**

Let $x, y \in N$ such that $f(x) = f(y)$

$$\Rightarrow x^2 + x + 1 = y^2 + y + 1$$

$$\Rightarrow (x - y)(x + y + 1) = 0$$

$$\Rightarrow x = y \text{ or } x = (-y - 1) \notin N$$

$\therefore f$ one-one.

Also, f is not onto.

PE

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	C	B	D	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	A	B	B	A	A	C

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