

Topic :-RELATIONS AND FUNCTIONS

1 (a)

We observe that the periods of $\sin x$ and $\sin \frac{x}{n}$ are $\frac{2\pi}{|n|}$ and $2|n|\pi$ respectively

Therefore, $f(x)$ is periodic with period $2|n|\pi$

But, $f(x)$ has period 4π

$$\therefore 2|n|\pi = 4\pi \Rightarrow |n| = 2 \Rightarrow n = \pm 2$$

2 (b)

It can be easily checked that $f:R \rightarrow R$ given by $f(x) = \log_a(x + \sqrt{x^2 + 1})$ is a bijection

Now, $f(f^{-1}(x)) = x$

$$\Rightarrow \log_a \left(f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1} \right) = x$$

$$\Rightarrow f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1} = a^x \quad \dots(i)$$

$$\Rightarrow \frac{1}{f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1}} = a^{-x}$$

$$\Rightarrow -f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1} = a^{-x} \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$2f^{-1}(x) = a^x - a^{-x}$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2}(a^x - a^{-x})$$

3 (d)

We have,

$$f(x) = x \frac{1 + \frac{2}{\sqrt{x+4}}}{2 - \sqrt{x+4}} + \sqrt{x+4} + 4\sqrt{x+4}$$

Clearly, $f(x)$ is defined for $x+4 > 0$ and $x \neq 0$

So, Domain of $f(x)$ is $(-4, 0) \cup (0, \infty)$

4 (d)

$$\therefore f(f(x)) = f\left(\frac{\alpha x}{x+1}\right)$$

$$= \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x \quad \text{[given]}$$

$$\Rightarrow \alpha^2 = \alpha x + x + 1$$

$$\Rightarrow \alpha^2 - 1 = (\alpha + 1)x$$

$$\Rightarrow (\alpha + 1)(\alpha - 1 - x) = 0$$

$$\Rightarrow \alpha + 1 = 0 \Rightarrow \alpha = -1 \quad [\because \alpha - 1 - x \neq 0]$$

5 **(d)**

$$f(x) = \operatorname{cosec}^2 3x + \cot 4x$$

Period of $\operatorname{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

\therefore Period of $f(x) = \operatorname{LCM}$ of $\left\{\frac{\pi}{3} \text{ and } \frac{\pi}{4}\right\}$

$$= \frac{\operatorname{LCM} \text{ of } (\pi, \pi)}{\operatorname{HCF} \text{ of } (3, 4)} = \frac{\pi}{1} = \pi$$

6 **(b)**

$$\text{Given, } f(x) = \sqrt{1 + \log_e(1-x)}$$

For domain, $(1-x) > 0$ and $\log_e(1-x) \geq -1$

$$\Rightarrow x < 1 \text{ and } 1-x \geq e^{-1}$$

$$\Rightarrow x < 1 \text{ and } x \leq 1 - \frac{1}{e}$$

$$\Rightarrow -\infty < x \leq \frac{e-1}{e}$$

7 **(d)**

$$\sin(\sin^{-1} x + \cos^{-1} x) = \sin\left(\frac{\pi}{2}\right) = 1$$

\therefore Range of $\sin(\sin^{-1} x + \cos^{-1} x)$ is 1.

8 **(d)**

$$\text{Given, } f(x) = \cos x - \sin x$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \right)$$

$$= \sqrt{2} \cos\left(\frac{\pi}{4} + x\right)$$

$$\text{Since, } -1 \leq \cos x \leq 1 \Rightarrow -1 \leq \cos\left(\frac{\pi}{4} + x\right) \leq 1$$

$$\Rightarrow -\sqrt{2} \leq \sqrt{2} \cos\left(\frac{\pi}{4} + x\right) \leq \sqrt{2}$$

\therefore Range is $[-\sqrt{2}, \sqrt{2}]$

9 **(a)**

$$\text{Given, } f(x) = x^2 + \frac{1}{x^2 + 1}$$

$$= (x^2 + 1) - \left(\frac{x^2}{x^2 + 1}\right)$$

$$= 1 + x^2 \left(1 - \frac{1}{x^2 + 1}\right) \geq 1, \forall x \in R$$

Hence, range of $f(x)$ is $[1, \infty)$.

10 (b)

Let $y = \sqrt{\sin 2x} \Rightarrow 0 \leq \sin 2x \leq 1$,

$$\Rightarrow 0 \leq 2x \leq \frac{\pi}{2}$$

$$\Rightarrow 0 \leq x \leq \frac{\pi}{4}$$

$$\Rightarrow x \in \left[n\pi, n\pi + \frac{\pi}{4} \right]$$

11 (c)

We have, $f(x) = x - [x] - \frac{1}{2}$

$$\therefore f(x) = \frac{1}{2} \Rightarrow x - [x] = 1$$

But, for any $x \in R$, $0 \leq x - [x] < 1$

$\therefore x - [x] \neq 1$ for any $x \in R$

$$\text{Hence, } \left\{ x \in R : f(x) = \frac{1}{2} \right\} = \phi$$

12 (c)

Since, $x \in [-2, 2]$, $x \leq 0$ and $f(|x|) = x$

For $-2 \leq x \leq 0$

$$f(-x) = x \Rightarrow (-x) - 1 = x \Rightarrow x = -\frac{1}{2}$$

13 (d)

Given, $f(x) = \sin x$

And $g(x) = \sqrt{x^2 - 1}$

\therefore Range of $f = [-1, 1] \notin$ domain of $g = (1, \infty)$

$\therefore gof$ is not defined.

14 (d)

Given, $f: C \rightarrow R$ such that $f(z) = |z|$

We know modulus of z and \bar{z} have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

15 (b)

We have,

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{2x}{-2} \text{ [Applying componendo-dividendo]}$$

$$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$$

$$\therefore f(2x) = \frac{2x-1}{2x+1} = \frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\}-1}{2\left\{\frac{f(x)+1}{1-f(x)}\right\}+1} = \frac{3f(x)+1}{f(x)+3}$$

16 (b)

Given, $f(x) = \tan \sqrt{\frac{\pi}{9} - x^2}$

For $f(x)$ to be defined $\frac{\pi^2}{9} - x^2 \geq 0$

$$\Rightarrow x^2 \leq \frac{\pi^2}{9} \Rightarrow -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$$

$$\therefore \text{Domain of } f = \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$

The greatest value of $f(x) = \tan \sqrt{\frac{\pi^2}{9} - 0}$, when $x = 0$

And the least value of $f(x) = \tan \sqrt{\frac{\pi^2}{9} - \frac{\pi^2}{9}}$, when $x = \frac{\pi}{3}$

\therefore The greatest value of $f(x) = \sqrt{3}$ and the least value of $f(x) = 0$

\therefore Range of $f = [0, \sqrt{3}]$.

17 **(b)**

We have,

$$[\sin x] = \begin{cases} 0, & 0 \leq x < \pi/2 \\ 1, & x = \pi/2 \\ 0, & \pi/2 < x \leq \pi \\ -1, & \pi < x < 2\pi \\ 0, & x = 2\pi \end{cases}$$

And, $\text{cosec}^{-1}x$ is defined for $x \in (-\infty, -1] \cup [1, \infty)$

$\therefore f(x) = \text{cosec}^{-1}[\sin x]$ is defined for $x = \frac{\pi}{2}$ and $x \in (\pi, 2\pi)$

Hence, domain of $\text{cosec}^{-1}[\sin x]$ is $(\pi, 2\pi) \cup \left\{\frac{\pi}{2}\right\}$

18 **(a)**

aRa if $|a - a| = 0 < 1$, which is true.

\therefore It is reflexive.

Now, aRb ,

$$|a - b| \leq 1 \Rightarrow |b - a| \leq 1$$

$$\Rightarrow aRb \Rightarrow bRa$$

\therefore It is symmetric.

19 **(b)**

Given

$$f(x) = \log_e(x - [x]) = \log_e\{x\}$$

When x is an integer, then the function is not defined.

\therefore Domain of the function $R - Z$.

20 **(b)**

Here, $f: [0, \infty) \rightarrow [0, \infty)$ i.e., domain is $[0, \infty)$ and codomain is $[0, \infty)$.

$$\text{For one-one } f(x) = \frac{x}{1+x}$$

$$\Rightarrow f'(x) = \frac{1}{(1+x)^2} > 0, \forall x \in [0, \infty)$$

$\therefore f(x)$ is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto (we find range)

$$f(x) = \frac{x}{1+x} \text{ i.e., } y = \frac{x}{1+x} \Rightarrow y + yx = x$$

$$\Rightarrow x = \frac{y}{1-y} \Rightarrow \frac{y}{1-y} \geq 0 \text{ as } x \geq 0 \therefore 0 \leq y \neq 1$$

ie, Range \neq Codomain

$\therefore f(x)$ is one-one but not onto.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	D	D	B	D	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	D	D	B	B	B	A	B	B

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