

Topic :-RELATIONS AND FUNCTIONS

1 (a)

Given, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$... (i)

Replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1 \quad \dots \text{(ii)}$$

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and subtracting Eq. (i) from Eq. (ii), we get

$$\begin{aligned} 5f(x^2) &= \frac{3}{x^2} - 1 - 2x^2 \\ \Rightarrow f(x^2) &= \frac{1}{5x^2} (3 - x^2 - 2x^4) \\ \Rightarrow f(x^4) &= \frac{1}{5x^4} (3 - x^4 - 2x^8) \quad [\text{Replacing } x \text{ by } x^2] \\ &= \frac{(1 - x^4)(2x^4 + 3)}{5x^4} \end{aligned}$$

2 (c)

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i) $7 - x \geq 0$ (ii) $x - 3 \geq 0$ (iii) $7 - x \geq x - 3$

Now,

$$\left. \begin{aligned} 7 - x \geq 0 &\Rightarrow x \leq 7 \\ x - 3 \geq 0 &\Rightarrow x \geq 3 \\ 7 - x \geq x - 3 &\Rightarrow x \leq 5 \end{aligned} \right\} \Rightarrow 3 \leq x \leq 5$$

Hence, the required domain is $\{3, 4, 5\}$

3 (a)

We have,

$f(x) = x$, $g(x) = |x|$ for all $x \in R$ and $\phi(x)$ satisfies the relation

$$[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$$

$$\Rightarrow \phi(x) - f(x) = 0 \text{ and } \phi(x) - g(x) = 0$$

$$\Rightarrow \phi(x) = f(x) \text{ and } \phi(x) = g(x)$$

$$\Rightarrow f(x) = g(x) = \phi(x)$$

But, $f(x) = g(x) = x$, for all $x \geq 0$ [$\because |x| = x$ for all $x \geq 0$]

$\therefore \phi(x) = x$ for all $x \in [0, \infty)$

4 (b)

We observe that $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ exists for

$$\frac{\pi^2}{16} - x^2 \geq 0 \Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

The least value of $\frac{\pi^2}{16} - x^2$ is 0 for $x = \pm \frac{\pi}{4}$ and the greatest value is $\frac{\pi^2}{16}$ for $x = 0$. Therefore, the greatest value of $f(x)$ occurs at $x = 0$ and the least value occurs at $x = \pm \pi/4$

Thus, greatest and least values of $f(x)$ are

$$f(0) = 3 \sin\left(\sqrt{\frac{\pi^2}{16}}\right) = 3 \sin \frac{\pi}{4} = \frac{3}{\sqrt{2}} \text{ and } f\left(\frac{\pi}{4}\right) = 3 \sin 0 = 0$$

Hence, the value of $f(x)$ lie in the interval $[0, 3/\sqrt{2}]$

ALITER For $x \in [-\pi/4, \pi/4] = \text{Dom}(f)$, we find that $\sqrt{\frac{\pi^2}{16} - x^2} \in [0, \pi/4]$

Since $\sin x$ is an increasing function on $[0, \pi/4]$

$$\therefore \sin x \leq \sin \sqrt{\frac{\pi^2}{16} - x^2} \leq \sin \pi/4$$

$$\Rightarrow 0 \leq 3 \sin \sqrt{\frac{\pi^2}{16} - x^2} \leq \frac{3}{\sqrt{2}} \Rightarrow 0 \leq f(x) \leq \frac{3}{\sqrt{2}}$$

5 (b)

$$f\left(\frac{\pi}{2} + x\right) = \left| \sin\left(\frac{\pi}{2} + x\right) \right| + \left| \cos\left(\frac{\pi}{2} + x\right) \right| \\ = |\cos x| + |\sin x| \text{ for all } x.$$

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

6 (d)

It can be easily checked that $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$ satisfies the relation $f \circ g(x) = g \circ f(x)$

7 (a)

Since, $(1, 2) \in S$ but $(2, 1) \notin S$

$\therefore S$ is not symmetric.

Hence, S is not an equivalent relation.

Given, $T = \{(x, y) : (x - y) \in I\}$

Now, $xTx \Rightarrow x - x = 0 \in I$, it is reflexive relation

Again, $xTy \Rightarrow (x - y) \in I$

$\Rightarrow y - x \in I \Rightarrow yTx$ it is symmetric relation.

Let xTy and yTz

$$\therefore x - y = I_1 \text{ and } y - z = I_2$$

Now, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$$\Rightarrow x - z \in I$$

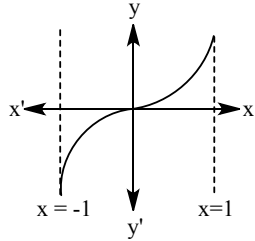
$$\Rightarrow xTz$$

$\therefore T$ is transitive.

Hence, T is an equivalent relation.

8 (d)

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

\therefore Function is one-one onto.

9 (c)

We have,

$$f(x) = \frac{1-x}{1+x}$$

$$\Rightarrow f(f(x)) = f\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x$$

Again,

$$f(x) = \frac{1-x}{1+x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{x-1}{x+1}$$

$$\therefore f\left(f\left(\frac{1}{x}\right)\right) = f\left(\frac{x-1}{x+1}\right) = \frac{1 - \frac{x-1}{x+1}}{1 + \frac{x-1}{x+1}} = \frac{1}{x}$$

$$\therefore \alpha = f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right) = x + \frac{1}{x}$$

$$\Rightarrow |\alpha| = \left|x + \frac{1}{x}\right| \geq 2$$

10 (b)

Let $A = \{1, 2, 3\}$

Let two transitive relations on the set A are

$$R = \{(1, 1), (1, 2)\}$$

And $S = \{(2, 2), (2, 3)\}$

Now, $R \cup S = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$

Here, $(1, 2), (2, 3) \in R \cup S \Rightarrow (1, 3) \notin R \cup S$

$\therefore R \cup S$ is not transitive.

11 (c)

$$f(1) = 3, f(2) = 4, f(3) = 5, f(4) = 6$$

$\Rightarrow 1 \in B, 2 \in B$ do not have any pre-image in A

$\Rightarrow f$ is one-one and into

12 **(b)**

We observe that

$|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ is true, if

$f(x) \geq 0$ and $\phi(x) \geq 0$

OR

$f(x) < 0$ and $\phi(x) < 0$

$\Rightarrow (x > -1 \text{ and } x > 2) \text{ or } (x < -1 \text{ and } x < 2)$

$\Rightarrow x \in (2, \infty) \cup (-\infty, -1)$

13 **(b)**

We have, $f(x) = \frac{\sin^{-1}(3-x)}{\log_e(|x|-2)}$

$\sin^{-1}(3-x)$ is defined for all x satisfying

$-1 \leq 3-x \leq 1 \Rightarrow -4 \leq -x \leq -2 \Rightarrow x \in [2, 4]$

$\log_e(|x|-2)$ is defined for all x satisfying

$|x|-2 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

Also, $\log_e(|x|-2) = 0$ when $|x|-2 = 1$ i.e., $x = \pm 3$

Hence, domain of $f = (2, 3) \cup (3, 4]$

14 **(a)**

$f(x)$ is defined

When $|x| > x$

$\Rightarrow x < -x, x > x$

$\Rightarrow 2x < 0, (x > x \text{ is not possible})$

$\Rightarrow x < 0$

Hence domain of $f(x)$ is $(-\infty, 0)$.

15 **(d)**

We have,

$f(x) = \log_{10}\{(\log_{10} x)^2 - 5(\log_{10} x) + 6\}$

Clearly, $f(x)$ assumes real values, if

$(\log_{10} x)^2 - 5 \log_{10} x + 6 > 0$ and $x > 0$

$\Rightarrow (\log_{10} x - 2)(\log_{10} x - 3) > 0$ and $x > 0$

$\Rightarrow (\log_{10} x < 2 \text{ or } \log_{10} x > 3) \text{ and } x > 0$

$\Rightarrow (x < 10^2 \text{ or } x > 10^3) \text{ and } x > 0 \Rightarrow x \in (0, 10^2) \cup (10^3, \infty)$

16 **(b)**

We have,

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow f(y) = y^2 - 2$, where $y = x + \frac{1}{x}$

Now,

$x > 0 \Rightarrow y = x + \frac{1}{x} \geq 2$ and, $x < 0 \Rightarrow y = x + \frac{1}{x} \leq -2$

Thus, $f(y) = y^2 - 2$ for all y satisfying $|y| \geq 2$

17 (c)

Since $\sin x$ is a periodic function with period 2π and

$$f(x) = \sin\left(\frac{2x+3}{6\pi}\right) = \sin\left(\frac{x}{3\pi} + \frac{1}{2\pi}\right)$$

$$\therefore f(x) \text{ is periodic with period } = \frac{2\pi}{1/3\pi} = 6\pi^2$$

18 (c)

Let $f(x) = y$. Then,

$$10x - 7 = y \Rightarrow x = \frac{y+7}{10} \Rightarrow f^{-1}(y) = \frac{y+7}{10}$$

$$\text{Hence, } f^{-1}(x) = \frac{x+7}{10}$$

19 (b)

$$\therefore f(2.5) = [2.5 - 2] = [0.5] = 0$$

20 (c)

We have,

$$f(x) = \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$$

Clearly, $f(x)$ assumes real values, if

$$\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3 \geq 0$$

$$\Rightarrow \log_{10} \left\{ \frac{\log_{10} x}{3(4 - \log_{10} x)} \right\} \geq 0$$

$$\Rightarrow \frac{\log_{10} x}{3(4 - \log_{10} x)} \geq 1$$

$$\Rightarrow \frac{4 \log_{10} x - 12}{3(4 - \log_{10} x)} \geq 0$$

$$\Rightarrow \frac{\log_{10} x - 3}{\log_{10} x - 4} \leq 0$$

$$\Rightarrow 3 \leq \log_{10} x < 4 \Rightarrow 10^3 \leq x < 10^4 \Rightarrow x \in [10^3, 10^4)$$

Hence, domain of $f = [10^3, 10^4)$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	B	B	D	A	D	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	A	D	B	C	C	B	C

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