DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :5

Topic :-RELATIONS AND FUNCTIONS

1 **(a)**

Here, $Y=\{7, 11, \ldots,\infty\}$

Let $y=4x+3 \Rightarrow \frac{y-3}{4}$

Inverse of $f(x)$ is

$$g(y)=\frac{y-3}{4}$$

2 **(b)**

We have,

$f(x)=\sqrt{\cos(\sin x)}+\sqrt{\sin(\cos x)}$

We observe that $f(x)$ is not defined in $(\pi/2, 3\pi/2)$ and it is aperiodic function with period 2π. So, let us consider the internal $[-\pi/2, \pi/2]$ as it domain. Further, since $f(x)$ is an even function. So, we will consider $f(x)$ defined on $[0, \pi/2]$ only.

Clearly, $\sqrt{\cos(\sin x)}$ and $\sqrt{\sin(\cos x)}$ are decreasing functions on $[0, \pi/2]$

Range $(f)=\left[f\left(\frac{\pi}{2}\right), f(0)\right]=[\sqrt{\cos 1}, 1+\sqrt{\sin 1}]$

4 **(c)**

We have,

$\log x>1$ for all $x \in (e,\infty)$

$\Rightarrow \log(\log x)>0$ for all $x \in (e,\infty)$

$\Rightarrow f(x)-\log[\log(\log x)] \in (-\infty,\infty)$ for all $x \in (e, \infty)$

Also, f is one-one. Hence, f is both one-one and onto

5 **(a)**

Given, $f(x)=x^2-3$

Now, $f(-1)=(-1)^2-3=-2$

$\Rightarrow fof(-1)=f(-2)=(-2)^2-3=1$

$\Rightarrow fofof(-1)=f(1)=1^2-3=-2$

Now, $f(0)=0^2-3=-3$

$\Rightarrow fof(0)=f(-3)=(-3)^2-3=6$

$\Rightarrow fof(0)=f(6)=6^2-3=33$

Again, $f(1)=1^2-3=-2$

$\Rightarrow fof(1)=f(-2)=(-2)^2-3=1$

$\Rightarrow fofof(-1)+fofof(0)+fofof(1)=-2+33-2=29$

Now, $f(4\sqrt{2}) = (4\sqrt{2})^2 - 3 = 32 - 3 = 29$

6 **(b)**

For any $x, y \in R$, we observe that

$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$

So, f is one-one

Let $\alpha \in R$ such that $f(x) = \alpha$

$\Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m-n\alpha}{1-\alpha}$

Clearly, $x \in R$ for $\alpha = 1$. So, f is not onto

Hence, f is one-one into. This fact can also be observed from the graph of the function

7 **(b)**

We have,

$D(f) = R$ and $D(g) = R - \{0\}$

$\therefore D(h) = R - \{0\}$

Hence, $h(x) = f(x)g(x) = x \times \frac{1}{x} = 1$ for all $x \in R - \{0\}$

8 **(b)**

Since $\cos\sqrt{x}$ is not a periodic function. Therefore, $f(x) = \cos\sqrt{x} + \cos^2 x$ is not a periodic function

9 **(b)**

We have, $f(x) = 2^x$

$\therefore \frac{f(n+1)}{f(n)} = \frac{2^{n+1}}{2^n} = 2$ for all $n \in N$

Hence, $f(0), f(1), f(2), \ldots$ are in G.P.

10 **(d)**

We have,

$f(\sin x) - f(-\sin x) = x^2 - 1$ for all $x \in R$ ...(i)

Replacing x by $-x$, we get

$f(-\sin x) - f(\sin x) = x^2 - 1$ ...(ii)

Adding (i) and (ii), we get

$2(x^2 - 1) = 0 \Rightarrow x = \pm 1$

$\therefore x^2 - 2 = 1 - 2 = -1$

12 **(d)**

For $f(x)$ to be defined

$-1 \le \log_2 x \le 1$ $[\because -1 \le \sin^{-1} x \le 1]$

$\Rightarrow \frac{1}{2} \le x \le 2$

13 **(a)**

We have,

$f(x) = |x|$ and $g(x) = [x]$

$\therefore g(f(x)) \le f(g(x))$

$\Rightarrow g(|x|) \le f([x]) \Rightarrow [|x|] \le |[x]|$

Clearly, $[|x|] = |[x]|$ for all $x \in Z$

Let $x \in (-\infty, 0)$ such that $x \notin Z$. Then, there exists positive integer k such that

$-k-1<x<-k$

$\Rightarrow [x]=-k-1$ and $k<|x|<k+1$

$\Rightarrow |[x]|=k+1$ and $[|x|]=k$

$\Rightarrow [|x|]<|[x]|$

Hence, $[|x|]\leq ||x||$ for all $x\in Z\cup(-\infty,0)$

i.e. $\{x\in R: g(f(x))\leq f(g(x))\}=Z\cup(-\infty,0)$

14 **(d)**

$$\therefore\ f\left(\frac{3x+x^3}{1+3x^2}\right)-f\left(\frac{2x}{1+x^2}\right)$$

$$=\log\left(\frac{1+\left(\frac{3x+x^3}{1+3x^2}\right)}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right)-\log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$=\log\left(\frac{1+x}{1-x}\right)^3-\log\left(\frac{1+x}{1-x}\right)^2$$

$$=\log\left(\frac{1+x}{1-x}\right)=f(x)$$

15 **(d)**

Clearly, $f(x)$ is defined if

$$=\underbrace{\log_{10}\log_{10}\ldots\log_{10}}_{\rightarrow(n-1)\text{ times}\leftarrow}x>0$$

$$\Rightarrow \underbrace{\log_{10}\ \log_{10}\ldots\log_{10}}_{(n-2)\text{ times}}\ x>1$$

$$\Rightarrow \underbrace{\log_{10}\ \log_{10}\ldots\log_{10}}_{(n-3)\text{ times}}\ x>10$$

$$\Rightarrow x>10^{10^{10^{\cdot^{\cdot^{(n-2)\text{ times}}}}}}$$

Thus, domain of $f=\left(10^{10^{10^{\cdot^{\cdot^{(n-2)\text{ times}}}}}},\infty\right)$

16 **(a)**

Let $y=\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$$\Rightarrow\ -1\leq\log_3\left(\frac{x}{3}\right)\leq 1$$

$$\Rightarrow\ \frac{1}{3}\leq\frac{x}{3}\leq 3$$

$$\Rightarrow\ 1\leq x\leq 9$$

17 **(d)**

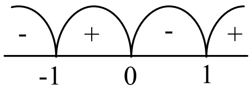
Since, $f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$

For domain of $f(x)$,

$x^3-1>0,\ 4-x^2\neq 0)$

$\Rightarrow\ x(x-1)(x+1)>0$ and $x\neq\pm 2$

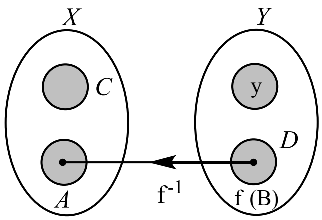
$\Rightarrow\ x \in (-1, 0) \cup (1, \infty), \quad x \neq \pm 2$



$\Rightarrow\ x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

18 **(c)**

The given data is shown in the figure below



Since, $f^{-1}(D) = x$

$\Rightarrow \quad f(x) = D$

Now, if $B \subset X, f(B) \subset D$

$\Rightarrow \quad f^{-1}(f(B)) = B$

19 **(b)**

Clearly, $f(x)$ is an odd function

20 **(c)**

We have,

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 \leq x \leq 2 \end{cases}$$

$\therefore f(|x|) = x \qquad [\because x \leq 0]$

$\Rightarrow f(-x) = x$

$\Rightarrow -x - 1 = x \Rightarrow x = -\frac{1}{2}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	C	A	B	B	B	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	A	D	D	A	D	C	B	C