

Topic :-RELATIONS AND FUNCTIONS

1 (d)

We observe that

Period of $\sin \frac{\pi x}{2}$ is $\frac{2\pi}{\pi/2} = 4$, Period of $\cos \frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3} = 6$, and,

Period of $\tan \frac{\pi x}{4}$ is $\frac{\pi}{\pi/4} = 4$

\therefore Period of $f(x) = \text{LCM of } (4, 6, 4) = 12$

2 (c)

We have,

$$f(x) = \lim_{x \rightarrow \infty} \frac{x^n + x^{-n}}{x^n + x^{-n}}$$

$$\Rightarrow f(x) = \lim_{x \rightarrow \infty} \frac{x^{2n} - 1}{x^{2n} + 1} = \frac{0 - 1}{0 + 1} = -1, \text{ if } -1 < x < 1$$

If $|x| > 1$, then $x^{2n} \rightarrow \infty$ as $n \rightarrow \infty$

$$\therefore f(x) = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^{2n}}}{1 + \frac{1}{x^{2n}}} = \frac{1 - 0}{1 + 1} = 1, \text{ if } |x| > 1$$

If $|x| = 1$, then $x^{2n} = 1$

$$\therefore f(x) = \lim_{x \rightarrow \infty} \frac{x^{2n} - 1}{x^{2n} + 1} = \frac{1 - 1}{1 + 1} = 0$$

Thus, we have

$$f(x) = \begin{cases} -1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| = 1 \\ 1, & \text{if } |x| > 1 \end{cases}$$

3 (c)

$R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ is a relation on

$A = \{1, 2, 3, 4\}$, then

(a) since, $(2, 4) \in R$ and $(2, 3) \in R$, so R is not a function.

(b) since, $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

So, R is not transitive.

(c) since, $(2, 3) \in R$ but $(3, 2) \notin R$, so R is not symmetric.

(d) since, $(4, 4) \notin R$, so R is not reflexive.

4 (a)

We have,

$$f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$$

Clearly, $f(x)$ is defined, if

$$16 - x \geq 2x - 1 > 0, 20 - 3x \geq 4x - 5 > 0 \text{ and } x \in Z$$

$$\Rightarrow x \in \{1, 2, 3, 4, 5\}, x \in \{2, 3\} \text{ and } x \in Z$$

$$\Rightarrow x \in \{2, 3\}$$

$$\therefore \text{Domain}(f) = \{2, 3\}$$

5 **(d)**

Given, $f(x) = e^{2ix}$ and $f: R \rightarrow C$. Function $f(x)$ is not one-one, because after some values of x (ie, π) it will give the same values.

Also, $f(x)$ is not onto, because it has minimum and maximum values $-1 - i$ and $1 + i$ respectively.

6 **(a)**

For $f(x)$ to be defined,

$$x - 4 \geq 0 \text{ and } 6 - x \geq 0 \Rightarrow x \geq 4 \text{ and } x \leq 6$$

Therefore, the domain is $[4, 6]$.

7 **(d)**

We have,

$$hogof(x) = \cos^{-1}(|\sin x|)$$

$$\text{and, } fogoh(x) = \sin^2(\sqrt{\cos^{-1}x})$$

Clearly, $hogof(x) \neq fogoh(x)$

Thus, option (a) is not correct

Now,

$$gofoh(x) = |\sin(\cos^{-1}x)| = |\sin(\sin^{-1}\sqrt{1-x^2})| = \sqrt{1-x^2}$$

$$\text{and, } fohog(x) = \sin^2(\cos^{-1}\sqrt{x}) = 1 - \cos^2(\cos^{-1}\sqrt{x})$$

$$\Rightarrow fohog(x) = 1 - \{\cos(\cos^{-1}\sqrt{x})\}^2 = 1 - x$$

$$\therefore gofoh(x) \neq fohog(x)$$

Thus, option (b) is correct

Also,

$$hogof(x) = \cos^{-1}(|\sin x|) \text{ and, } fohog(x) = 1 - x$$

$$\therefore hogof(x) \neq fohog(x)$$

Thus, option (c) is not correct

Hence, option (d) is correct

8 **(a)**

We have,

$$f(x) = \frac{2^x + 2^{-x}}{2}$$

$$\therefore f(x+y)f(x-y) = \frac{2^{x+y} + 2^{-x-y}}{2} \times \frac{2^{x-y} + 2^{-x+y}}{2}$$

$$\Rightarrow f(x+y)f(x-y) = \frac{2^{2x} + 2^{-2y} + 2^{2y} + 2^{-2x}}{4}$$

$$\Rightarrow f(x+y)f(x-y) = \frac{1}{2} \left(\frac{2^{2x} + 2^{-2x}}{2} + \frac{2^{2y} + 2^{-2y}}{2} \right)$$

$$\Rightarrow f(x+y) f(x-y) = \frac{1}{2} \{f(2x) + f(2y)\}$$

9 (b)

$$\begin{aligned} R &= \{(a, b) : a, b \in \mathbb{N}, a - b = 3\} \\ &= \{(n+3, n) : n \in \mathbb{N}\} \\ &= \{(4, 1), (5, 2), (6, 3), \dots\} \end{aligned}$$

10 (a)

Clearly, $f(x) = \sin^{-1} \left\{ \log_3 \left(\frac{x}{3} \right) \right\}$ exists if

$$-1 \leq \log_3 \left(\frac{x}{3} \right) \leq 1 \Leftrightarrow 3^{-1} \leq \frac{x}{3} \leq 3^1 \Leftrightarrow 1 \leq x \leq 9$$

Hence, domain of $f(x)$ is $[1, 9]$

11 (c)

For $f(x)$ to be defined, we must have

$$\frac{\sqrt{4-x^2}}{1-x} > 0, 4-x^2 > 0 \text{ and } 1-x \neq 0$$

$$\Rightarrow 1-x > 0, 4-x^2 > 0 \text{ and } 1-x \neq 0$$

$$\Rightarrow x < 1, x \in (-2, 2) \text{ and } x \neq 1 \Rightarrow x \in (-2, 1)$$

$$\therefore \text{Domain}(f) = (-2, 1)$$

Now, for $x \in (-2, 1)$, we have

$$-\infty < \log \left(\frac{\sqrt{4-x^2}}{1-x} \right) < \infty$$

$$\Rightarrow -1 \leq \sin \left\{ \log \left(\frac{\sqrt{4-x^2}}{1-x} \right) \right\} \leq 1 \Rightarrow -1 \leq f(x) \leq 1$$

Hence, Range $(f) = [-1, 1]$

12 (a)

Given, $f(x) = \frac{ax+b}{cx+d}$ and $f \circ f(x) = x$

$$\Rightarrow f \left(\frac{ax+b}{cx+d} \right) = x$$

$$\Rightarrow \frac{a \left(\frac{ax+b}{cx+d} \right) + b}{c \left(\frac{ax+b}{cx+d} \right) + d} = x$$

$$\Rightarrow \frac{x(a^2+bc) + ab+bd}{x(ac+cd) + bc+d^2} = x$$

$$\Rightarrow d = -a$$

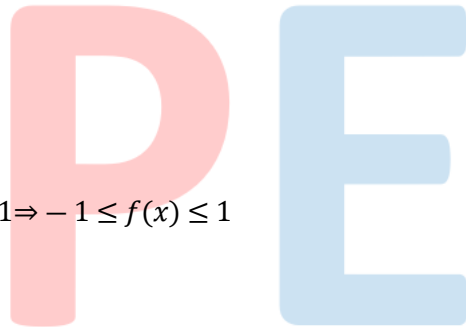
13 (c)

If $f: C \rightarrow C$ given by $f(x) = \frac{ax+b}{cx+d}$ is a constant function, then

$f(x) = \text{Constant} (= \lambda, \text{ say})$ for all $x \in C$

$$\Rightarrow \frac{ax+b}{cx+d} = \lambda \text{ for all } x \in C$$

$$\Rightarrow (a-\lambda c)x + (b-\lambda d) = 0 \text{ for all } x \in C$$



$$\Rightarrow a - \lambda c = 0 \text{ and } b - \lambda d = 0 \Rightarrow \frac{a}{c} = \frac{b}{d} \Rightarrow ad = bc$$

14 (d)

Periods of $\sin \lambda x + \cos \lambda x$ and $|\sin x| + |\cos x|$ are $\frac{2\pi}{\lambda}$ and $\frac{\pi}{2}$ respectively

$$\therefore \frac{\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 4$$

15 (b)

$$\text{We have, } f(x) = \sqrt{\log_{16} x^2}$$

Clearly, $f(x)$ exists, if

$$\log_{16} x^2 \geq 0 \Rightarrow x^2 \geq 1 \Leftrightarrow |x| \geq 1$$

16 (b)

Since, $f(x)$ is an even function, therefore $f'(x)$ is an odd function

$$\text{i.e., } f'(-e) = -f'(e)$$

$$\therefore f'(e) + f'(-e) = 0$$

17 (c)

We have,

$$f(x) = \log\left(\frac{1+x}{1-x}\right)$$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left\{\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right\} = \log\left(\frac{x+1}{1-x}\right)^2$$

$$\Rightarrow f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+x}{1-x}\right) = 2f(x)$$

18 (c)

$$f(x) = \cos^2 x + \sin^4 x = 1 - \cos^2 x + \cos^4 x$$

$$\Rightarrow f(x) = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} \text{ for all } x$$

$$\text{Also, } f(x) = \cos^2 x + \sin^4 x \leq \cos^2 x + \sin^2 x = 1$$

$$\therefore \text{Range } (f) = [3/4, 1]$$

$$\text{Hence, } f(R) = [3/4, 1]$$

19 (d)

For domain of given function

$$-1 \leq \log_2\left\{\frac{x}{12}\right\} \leq 1$$

$$\Rightarrow 2^{-1} \leq \frac{x}{12} \leq 2$$

$$\Rightarrow 6 \leq x \leq 24$$

$$\Rightarrow x \in [6, 24]$$

20 (d)

$$\text{Given, } f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

Here, 4^{-x^2} is defined for $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$, $\cos^{-1}\left(\frac{x}{2} - 1\right)$ is defined,

If $-1 \leq \frac{x}{2} - 1 \leq 1 \Rightarrow 0 \leq x \leq 4$

And $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Hence, $f(x)$ is defined for $x \in [0, \frac{\pi}{2}]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	A	D	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	D	B	B	C	C	D	D

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