DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-RELATIONS AND FUNCTIONS

1   **(c)**
Clearly, $X = R^+$ and $Y = R$

2   **(b)**
Given, $f(x).f\left(\frac{1}{2}\right) = f(x) + f\left(\frac{1}{x}\right)$
Let $f(x) = x^n \pm 1$, where $n \in I$.
Now, $f(4) = 65$
Case I
Let $f(x) = x^n + 1$
$\Rightarrow f(4) = 4^n + 1$
$\Rightarrow 65 = 4^n + 1$
$\Rightarrow n = 3$
Case II
Let $f(x) = x^n - 1$
$\Rightarrow f(4) = 4^n - 1 \Rightarrow 65 = 4^n - 1$
$\Rightarrow 4^n = 66$
The quality does not hold true for $n \in Z$.
Therefore, $f(x) = x^3 + 1$
Now, $f(6) = 6^3 + 1 = 216 + 1 = 217$

3   **(b)**
Since, the graph is symmetrical about the line$= x = 2$
$\Rightarrow f(2 + x) = f(2 - x)$

4   **(c)**
We have,
$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ and $g(x) = x(1 - x^2)$

$\therefore fog\ (x) = f(g(x))$
$\Rightarrow fog\ (x) = \begin{cases} -1, & \text{if } g(x) < 0 \\ 0, & \text{if } g(x) = 0 \\ 1, & \text{if } g(x) > 0 \end{cases}$
$\Rightarrow fog\ (x) = \begin{cases} -1, & \text{if } x \in (-1,0) \cup (1,\infty) \\ 0, & \text{if } x = 0, \pm 1 \\ 1, & \text{if } x \in (-\infty, -1) \cup (0, 1) \end{cases}$

5   **(b)**

Reflexive xRx

Since, $x^2 = x.x$

$\qquad x^2 = xy$

Transitive, $xRy \Rightarrow x^2 = xy$

And $yRz \Rightarrow y^2 = yz$

Now, $x^2 y^2 = xy^2 z \Rightarrow x^2 = xz$

$\Rightarrow xRz$

$\therefore$ It is transitive.

6   **(c)**

We have,

$$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right), n \in Z, n > 2$$

Since $\sin\left(\frac{\pi x}{n-1}\right)$ and $\cos\left(\frac{\pi x}{n}\right)$ are periodic functions with period $2(n-1)$ and $2n$ respectively.

Therefore, $f(x)$ is periodic with period equal to LCM of $(2n, 2(n-1)) = 2n(n-1)$

7   **(b)**

Let $g(x)$ be the even extension of $f(x)$ on $[-4, 4]$

Then,

$$g(x) = \begin{cases} f(x) \text{ for } x \in [-4, 0] \\ f(-x) \text{ for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} e^x + \sin x \text{ for } x \in [-4, 0] \\ e^{-x} + \sin(-x) \text{ for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} e^x + \sin x \text{ for } x \in [-4, 0] \\ e^{-x} - \sin x \text{ for } x \in [0, 4] \end{cases}$$

$$\Rightarrow g(x) = e^{-|x|} - \sin|x| \text{ for } x \in [-4, 4]$$

8   **(d)**

Clearly, $f(x)$ is an even function and $f(x) < 0$ for all $x > 0$

Therefore, the graph of $f(x)$ lies in the third and fourth quadrants

9   **(d)**

The given function is

$$f(x) = \sqrt{1 - 2x} + 2\sin^{-1}\left(\frac{3x-1}{2}\right)$$

For domain of $f(x)$, $1 - 2x \geq 0$ and $-1 \leq \frac{3x-1}{2} \leq 1$

$\Rightarrow \quad x \leq \frac{1}{2}$ and $-2 \leq 3x - 1 \leq 2$

$\Rightarrow \quad x \leq \frac{1}{2}$ and $-\frac{1}{3} \leq x \leq 1$

$\therefore$ Domain of $f(x) = \left[-\frac{1}{3}, \frac{1}{2}\right]$

10   **(c)**

We have,

$$f(x) = \log_{(x+3)}(x^2 - 1)$$

Clearly, $f(x)$ is defined for x satisfying the following conditions

(i) $x^2 - 1 > 0$ (ii) $x + 3 > 0$ and $x + 3 \neq 1$

Now, $x^2 - 1 > 0 \Rightarrow x \in (-\infty, -1) \cup (1, \infty)$
and,
$x + 3 > 0$ and $x + 3 \neq 1 \Rightarrow x > -3$ and $x = -2$
$\Rightarrow x \in (-3, -2) \cup (-2, \infty)$
Hence, the domain of $f(x)$ is $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

11   (b)
$x^2 - 6x + 7 = (x - 3)^2 - 2$
Obviously, minimum value is -2 and maximum is ∞.

12   (d)
We have,
$fof^{-1}(x) = x$
$\Rightarrow f(f^{-1}(x)) = x$
$\Rightarrow f(y) = x$ where $y = f^{-1}(x)$
$\Rightarrow \dfrac{e^y - e^{-y}}{e^y + e^{-y}} + 2 = x \Rightarrow \dfrac{e^y - e^{-y}}{e^y + e^{-y}} = x - 2 \Rightarrow \dfrac{2e^y}{-2e^{-y}} = \dfrac{x-1}{x-3}$
$\Rightarrow e^{2y} = \dfrac{x-1}{3-x}$
$\Rightarrow y = \dfrac{1}{2} \log\left(\dfrac{x-1}{3-x}\right)$
$\Rightarrow f^{-1}(x) = \dfrac{1}{2} \log\left(\dfrac{x-1}{3-x}\right)$

13   (b)
$f(x) = \dfrac{4^x}{4^x + 2}$
$\therefore \quad f(1-x) + f(x) = \dfrac{4^{1-x}}{4^{1-x} + 2} + \dfrac{4^x}{4^x + 2}$
$= \dfrac{4}{4 + 2 \cdot 4^x} + \dfrac{4^x}{4^x + 2} = \dfrac{2}{2 + 4^x} + \dfrac{4^x}{4^x + 2} = 1$
By putting $x = \dfrac{1}{97}, \dfrac{2}{97}, \dfrac{3}{97}, \dots \dfrac{48}{97}$
And adding, we get
$f\left(\dfrac{1}{97}\right) + f\left(\dfrac{2}{97}\right) + \dots + f\left(\dfrac{96}{97}\right) = 48$
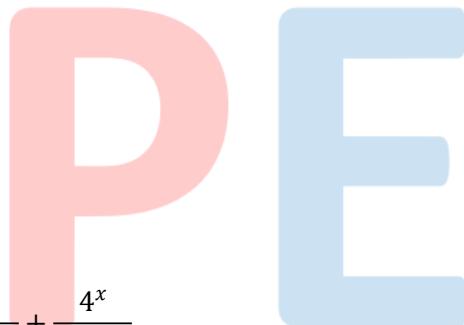
14   (c)
Given, $f(x) = \dfrac{2\sin 8x \cos x - 2\sin 6x \cos 3x}{2\cos 2x \cos x - 2\sin 3x \sin 4x}$
$= \dfrac{(\sin 9x + \sin 7x) + (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)}$
$= \dfrac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x}$
$= \dfrac{2 \cos 5x \sin 2x}{2 \cos 2x \cos 5x} = \tan 2x$
$\therefore$ Period of $f(x) = \dfrac{\pi}{2}$

15   **(d)**

$gof = g\{f(x)\} = g(x^2) = x^2 + 5$

16   **(b)**

We have,

$f(x) = \log_{2x-5}(x^2 - 3x - 10)$

For $f(x)$ to be defined, we must have

$x^2 - 3x - 10 > 0, 2x - 5 > 0$ and $2x - 5 \neq 1$

$\Rightarrow (x - 5)(x + 2) > 0, x > \dfrac{5}{2}$ and $\dfrac{5}{2}$ and $x \neq 3$

$\Rightarrow x > 5 \Rightarrow x \in (5, \infty)$

17   **(c)**

Since, $f(x)$ is an even function therefore its values is always greater than equal to 0 and we know

    $x^2 < x^2 + 1$ or $\dfrac{x^2}{x^2 + 1} < 1$

$\therefore$ Required range is $[0, 1)$.

18   **(d)**

We have,

$f(x^2) = |x^2 - 1| \neq |x - 1|^2 = [f(x)]^2$

$f(|x|) = ||x| - 1| \neq |x - 1| = |f(x)|$

And,

$f(x + y) = |x + y - 1| \neq |x - 1| + |y - 1| = f(x) + f(y)$

Hence, none of the above given option is true

19   **(d)**

We have,

$f(x + 2) - 2f(x + 1) + f(x)$

$= a^{x+2} - 2\,a^{x+1} + a^x$

$= a^x(a^2 - 2\,a + 1) = a^x(a - 1)^2 = (a - 1)^2 f(x)$

So, option (a) holds

It can be easily checked that options (b) and (c) are also true but option (d) is not true

20   **(b)**

It can be easily seen that $f{:}A{\to}A$ is a bijection. Let $f(x) = y$. Then,

$f(x) = y$

$\Rightarrow x(2 - x) = y$

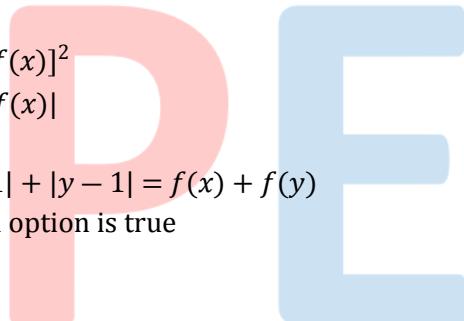$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4 - 4y}}{2}$

$\Rightarrow x = 1 \pm \sqrt{1 - y}$

$\Rightarrow x = 1 - \sqrt{1 - y} \qquad [\because x \leq 1]$

$\Rightarrow f^{-1}(y) = 1 - \sqrt{1 - y}$

Hence, $f^{-1}{:}A{\to}A$ is defined as $f^{-1}(x) = 1 - \sqrt{1 - x}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	B	C	B	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	D	B	C	D	D	B