

Topic :-PROBABILITY

1 (c)

$$\text{Given, } P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = \frac{7}{10}$$

$$\text{Since, } P(A \cap B) + P(\overline{A \cap B}) = 1$$

$$\Rightarrow P(A \cap B) = 1 - \frac{7}{10} = \frac{3}{10}$$

$$\text{Also, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{4}{5} = P(A) + \frac{2}{5} - \frac{3}{10}$$

$$\Rightarrow P(A) = \frac{4}{5} - \frac{2}{5} + \frac{3}{10}$$

$$= \frac{2}{3} + \frac{3}{10} = \frac{7}{10}$$

2 (c)

Probability of getting an ace

$$P(E_1) = \frac{4}{52} = \frac{1}{13}, P\left(\frac{E_2}{E_1}\right) = \frac{15}{51} = \frac{5}{17}$$

$$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$$

$$= \frac{1}{13} \cdot \frac{5}{17} = \frac{5}{221}$$

3 (b)

4 five-rupee, 3 two-rupee and 2 one-rupee coins can be stacked together in a column in $\frac{9!}{4!3!2!}$ ways

The number of ways in which coins of the same denomination the consecutive is same as the number of ways of arranging 3 distinct items i.e. 3! Ways

$$\text{Hence, required probability} = \frac{3!}{\frac{9!}{4!3!2!}} = \frac{1}{210}$$

4 (b)

The total number of ways of selecting two squares is 64×63

For each of the four corner squares, the favourable number of way is 2

For each of the 24 non-corner squares on either side of chess board, the favourable number of cases is 3

For each of the 36 remaining squares, the favourable, number of ways is 4

Thus, the total number of favourable ways

$$= 4 \times 2 + 24 \times 3 + 36 \times 4 = 224$$

$$\text{Hence, required probability} = \frac{224}{64 \times 63} = \frac{1}{18}$$

5 **(b)**

We define the following events:

A_1 : He knows the answer;

A_2 : He does not know the answer;

E : He gets the correct answer

$$\text{Then, } P(A_1) = \frac{9}{10}, P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}$$

$$P(E | A_1) = 1, P(E | A_2) = \frac{1}{4}$$

$$\therefore \text{ Required probability} = P(A_2 | E)$$

$$= \frac{P(A_2)P(E|A_2)}{P(A_1)P(E|A_1) + P(A_2)P(E|A_2)} = \frac{\frac{1}{10} \cdot \frac{1}{4}}{\frac{9}{10} \cdot 1 + \frac{1}{10} \cdot \frac{1}{4}} = \frac{1}{37}$$

6 **(d)**

$$P(A \cap B') = P(A) - P(A \cap B)$$

7 **(d)**

Let A = Event of getting i on first dice

and B = Event of getting more than i on second dice

$$\therefore \text{ Required probability} = \sum_{i=1}^5 P(A_i \cap B_1)$$

$$= \frac{1}{6} [P(B_1) + P(B_2) + P(B_3) + P(B_4) + P(B_5)]$$

$$= \frac{1}{6} \left[\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6} \right]$$

$$= \frac{15}{36} = \frac{5}{12}$$

8 **(a)**

The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$$\therefore \text{Probability of the required event} = \frac{1}{2}$$

9 (a)

Let A_1 denote the event that a coin having heads on both sides is chosen, and A_2 denote the event that a fair coin is chosen. Let E denote the event that head occurs. Then

$$P(A_1) = \frac{n}{2n+1}, P(A_2) = \frac{n+1}{2n+1}, P(E/A_1) = 1, P(E/A_2) = \frac{1}{2}$$

Now, $P(E) = P(A_1 \cap E) + P(A_2 \cap E)$

$$\Rightarrow P(E) = P(A_1)P(E/A_1) + P(A_2)P(E/A_2)$$

$$\Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

$$\Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)}$$

$$\Rightarrow 124n + 62 = 126n + 42$$

$$\Rightarrow 2n = 20 \Rightarrow n = 10$$

10 (d)

In binomial distribution, mean = $np = 10$, variance = $npq = 5$

$$\therefore p = q = \frac{1}{2}$$

Let x be the mode, then

$$np + p < x > np - q$$

$$\therefore 10 + \frac{1}{2} > x > 10 - \frac{1}{2}$$

$$\Rightarrow \frac{21}{2} > x > \frac{19}{2} \Rightarrow 9.5 < x < 10.5$$

$$\therefore x = 10$$

11 (b)

Mean = $np = 4$, variance = $nqp = 3$

On solving, we get $q = \frac{3}{4}$, $n = 16$, $p = \frac{1}{4}$

Now, $P(X \geq 1) = 1 - P(X = 0) = 1 - {}^nC_0 p^0 q^{n-0}$

$$= 1 - \left(\frac{3}{4}\right)^{16}$$

12 (c)

Probability of defective transistor = $\frac{5}{15} = \frac{1}{3}$ and probability of non-defective transistor = $1 - \frac{1}{3} = \frac{2}{3}$

Probability that the inspectors finds non-defective transistors

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

Hence, probability that atleast one of the inspectors finds a defective transistor

$$= 1 - \frac{8}{27} = \frac{19}{27}$$

13 **(b)**

The probability of suffering of a disease is 10%

$$p = \frac{10}{100} = \frac{1}{10} \text{ and } q = \frac{9}{10}$$

Total number of patients, $n = 6$

∴ Required probability

$$= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3$$

$$= \frac{6.5.4}{3.2.1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000}$$

$$= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5}$$

14 **(c)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore 0.6 = P(A) + P(B) - 0.2$$

$$\Rightarrow P(A) + P(B) = 0.8$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 12 \quad [\because P(A) = 1 - P(\bar{A})]$$

15 **(c)**

$$n(S) = 36$$

Let $E =$ Event of getting sum 7

$$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$\therefore n(E) = 6$$

$E =$ Event of getting sum 11

$$= \{(6,5), (5,6)\}$$

$$\therefore n(F) = 2$$

$$\text{Also } n(E \cap F) = 0$$

PE

$$\therefore n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

$$= 6 + 2 = 8$$

$$\therefore \text{Required probability} = \frac{8}{36} = \frac{2}{9}$$

16 (a)

Since the graph of $y = 16x^2 + 8(a+5)x - 7a - 5$ is strictly above x-axis. Therefore, $y > 0$ for all x

$$\Rightarrow 16x^2 + 8(a+5)x - 7a - 5 > 0 \text{ for all } x$$

$$\Rightarrow 64(a+5)^2 + 64(7a+5) < 0 \quad [\because \text{Disc} < 0]$$

$$\Rightarrow a^2 + 17a + 30 < 0$$

$$\Rightarrow -15 < a < -2$$

$$\therefore \text{Required probability} = \frac{\int_{-15}^{-2} dx}{\int_{-20}^0 dx} = \frac{13}{20}$$

17 (c)

Duplicate = 5, original = 10

Taking 3 times.

The probability that none of the items is duplicate i.e., all the three are original

$$= \frac{{}^{10}C_3}{{}^{15}C_3} = \frac{24}{91}$$

18 (b)

The probability that only two tests are needed = (probability that the first tested machine is faulty)

$$\times (\text{probability that the second tested machine is faulty given the first machine tested is faulty}) = \frac{2}{4}$$

$$\times \frac{1}{3} = \frac{1}{6}$$

19 (c)

Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways. Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$.

Therefore, $x_1, x_2 < 30$ i.e. x_1 and x_2 should come from tickets numbered 1 to 29 and this may happen

in ${}^{29}C_2$ ways. Remaining two i.e. $x_4, x_5 < 30$, should come from 20 tickets numbered 31 to 50 in ${}^{20}C_2$ ways.

C_2 ways.

So, favourable number of elementary events = ${}^{29}C_2 \times {}^{20}C_2$

$$\text{Hence, required probability} = \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$$

20 (a)

$$\text{Probability of no tail in four throws} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

$$\text{Probability of atleast one tail} = 1 - \frac{1}{16} = \frac{15}{16}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	B	B	B	D	D	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	C	C	A	C	B	C	A

PE