

## Topic :-PROBABILITY

1 (a)

Given, one integer is chosen at random from the first 200 positive integers and integer chosen is divisible by 6 or 8.

∴ One integer can be chosen out of 200 integers in  ${}^{200}C_1$  ways.

Let  $A$  be the event that an integer selected is divisible by 6 and  $B$  that it is divisible by 8.

$$\text{Then, } P(A) = \frac{33}{200}, P(B) = \frac{25}{200}$$

$$\text{and } P(A \cap B) = \frac{8}{200}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}$$

2 (c)

Given that,  $E(X) = 3$  and  $E(X^2) = 11$

Variance of  $X = E(X^2) - [E(X)]^2$

$$= 11 - (3)^2 = 11 - 9 = 2$$

3 (b)

Let  $X$  be the number of heads getting in  $n$  tossed. Therefore,  $X$  follows binomial distribution with parameters

$$n, p = \frac{1}{2}, q = \frac{1}{2}$$

Since,  $P(X \geq 1) \geq 0.8$  [given]

$$\therefore 1 - P(X = 0) \geq 0.8$$

$$\Rightarrow P(X = 0) \leq 0.2$$

$$\Rightarrow {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n \leq 0.2$$

$$\Rightarrow \frac{1}{2^n} \leq \frac{1}{5} \Rightarrow 2^n \geq 5$$

Hence, least value of  $n$  is 3

4 **(c)**

Total number of digits in any number at the unit place is 10.

$$\therefore n(S) = 10$$

To get the last digit in product is 1, 3, 5 or 7, it is necessary the last digit in each number must be 1, 3, 5 or 7.

$$n(A) = 4,$$

$$\therefore P(A) = \frac{4}{10} = \frac{2}{5}$$

$$\text{Hence, required probability} = \left(\frac{2}{5}\right)^4 = \frac{16}{625}$$

5 **(a)**

Since, events are independent, so

$$P(A \cap B') = P(A) \times P(B') = \frac{3}{25}$$

$$\Rightarrow P(A) \times [(1 - P(B))] = \frac{3}{25} \dots(i)$$

$$\text{Similarly, } P(B) \times [1 - P(A)] = \frac{8}{25} \dots(ii)$$

$$\therefore \text{From Eqs.(i) and (ii), } P(A) = \frac{1}{5}$$

6 **(a)**

We have,

$$x^2 - 3x + 2 \geq 0 \Rightarrow (x - 1)(x - 2) \geq 0 \Rightarrow x < 1 \text{ or } x > 2$$

$$\therefore \text{Required probability} = \frac{\int_0^1 dx + \int_2^5 dx}{\int_0^5 dx} = \frac{4}{5}$$

7 **(d)**

$$P(\text{At least on head}) = 1 - P(\text{zero head})$$

$$= 1 - P(\text{all three tails})$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

8 **(b)**

For binomial distribution

$$0 < \text{variance} < \text{mean}$$

$$\Rightarrow 0 < \beta < \alpha$$

9 **(c)**

$$\text{Given, } x^2 - n = 0$$

$$\Rightarrow x = \pm \sqrt{n}$$

$$\therefore n = 1, 4, 9, 16, 25, 36$$

$$\therefore \text{Required probability} = \frac{6}{40} = \frac{3}{20}$$

10 **(b)**

In a pack of 52 cards, there are 26 black cards.

$$\begin{aligned}\therefore \text{Required probability} &= \frac{{}^{26}C_3}{{}^{52}C_3} \\ &= \frac{26 \times 25 \times 24}{3 \times 2 \times 1} \times \frac{3 \times 2 \times 1}{52 \times 51 \times 50} \\ &= \frac{2}{17}\end{aligned}$$

11 **(d)**

We have,

$$\text{Total number of elementary events} = 6^3 = 216$$

Exactly two of three dice will show the same number in

$${}^6C_1 \times {}^5C_1 \times \frac{3!}{2!} \text{ ways}$$

$\therefore$  Favourable number of elementary events

$$= {}^6C_1 \times {}^5C_1 \times \frac{3!}{2!} = 90$$

$$\text{Hence, required probability} = \frac{90}{216}$$

12 **(c)**

Since,  $P(B) = \frac{2}{7}$  and  $P(A \cup B^c) = 0.8$

$$P(B^c) = 1 - \frac{2}{7} = \frac{5}{7}$$

$$\text{Using, } P(A \cup B^c) = P(A) + P(B^c) - P(A) \cdot P(B^c)$$

$$\Rightarrow 0.8 = P(A) + \frac{5}{7} - \frac{5}{7}P(A)$$

$$\Rightarrow 0.8 = \frac{5}{7} + \frac{2}{7}P(A)$$

$$\Rightarrow P(A) = 0.3$$

13 **(d)**

Clearly,  $P(A \cup B \cup C) = 1$

$$\Rightarrow P(A) + P(B) + P(C) = 1$$

$$\Rightarrow P(A) + \frac{1}{2}P(A) + \frac{1}{3}P(A) = 1$$

$$\Rightarrow \frac{11}{6}P(A) = 1$$

$$\Rightarrow P(A) = \frac{6}{11}$$

PE

14 (d)

$$P(X = 0) = k, P(X = 1) = 2k\left(\frac{1}{5}\right)^1$$

$$P(X = 2) = 3k\left(\frac{1}{5}\right)^2, \dots$$

Since,  $P(X = 0) + P(X = 1) + P(X = 2) + \dots = 1$

$$\therefore k + 2k\left(\frac{1}{5}\right) + 3k\left(\frac{1}{5}\right)^2 + \dots = 1$$

$$\text{and } k + 2k\left(\frac{1}{5}\right)^2 + \dots = \frac{1}{5}$$

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$$k + k\left(\frac{1}{5}\right) + k\left(\frac{1}{5}\right)^2 + \dots = \frac{4}{5}$$

$$\Rightarrow \frac{k}{1 - \frac{1}{5}} = \frac{4}{5}$$

$$\Rightarrow k = \frac{16}{25}$$

$$\therefore P(X = 0) = \frac{16}{25}(0 + 1)\left(\frac{1}{5}\right)^0 = \frac{16}{25}$$

15 (c)

$$\text{Given, } P(\bar{B}) = \frac{1}{3} \Rightarrow P(B) = \frac{2}{3}$$

$$P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$$

Now,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{5}{6} = P(A) + \frac{2}{3} - \frac{1}{3}$$

$$\Rightarrow P(A) = \frac{5}{6} - \frac{2}{3} + \frac{1}{3} = \frac{1}{2}$$

16 (c)

We have,

PE

Total number of ways of selecting 4 tickets =  $3^4 = 81$

Favourable number of ways

$$= \text{Sum of the coefficients of } x^2, x^4, \dots \text{ in } (x + x^2 + x^3)^4$$

$$= \text{Sum of the coefficients of } x^2, x^4, \dots \text{ in } x^4(1 + x + x^2)^4$$

$$\text{Let } (1 + x + x^2)^4 = 1 + a_1 x + a_2 x^2 + \dots + a_8 x^8$$

Putting  $x = 1$  and  $x = -1$  respectively, we get

$$3^4 = 1 + a_1 + a_2 + a_3 + \dots + a_8$$

$$\text{and, } 1 = 1 - a_1 + a_2 - a_3 + \dots + a_8$$

$$\therefore 3^4 + 1 = 2(1 + a_2 + a_4 + a_6 + a_8)$$

$$\Rightarrow a_2 + a_4 + a_6 + a_8 = 40$$

Thus, the sum of the coefficients of  $x^2, x^4, \dots = 40$

$$\text{Hence, required probability} = \frac{40}{81}$$

17 (c)

Probability of getting a white ball at any draw is,  $p$

$$= \frac{12}{24} = \frac{1}{2}$$

The probability of getting a white ball 4th in the 7th draw

$$= P(\text{getting 3 white balls in 6 draws}) \times P(\text{white ball at the 7th draw})$$

$$= {}^6C_3 \left(\frac{1}{2}\right)^6 \cdot \frac{1}{2} = \frac{20}{2^7} = \frac{5}{32}$$

18 (c)

Total number of persons = 15

and number of persons who can speak Hindi and English both

$$= 10 + 8 - 15 = 3$$

$$\therefore \text{Required probability} = \frac{{}^7C_1 \times {}^3C_1}{{}^{15}C_2} = \frac{7 \times 3}{\frac{15 \times 14}{2}} = \frac{1}{5}$$

19 (b)

$$\text{Required probability} = \frac{1}{2} \left( \frac{{}^3C_1}{{}^7C_1} + \frac{{}^2C_1}{{}^8C_1} \right)$$

$$= \frac{1}{2} \left( \frac{3}{7} + \frac{2}{8} \right) = \frac{19}{56}$$

20 (c)

Favourable cases will be (5,1), (4,2), (2,4), (1,5)

$$\text{Hence, required probability} = \frac{4}{6.5} = \frac{2}{15}$$

<b>ANSWER-KEY</b>										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	C	A	A	D	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	D	D	C	C	C	C	B	C

PE