

Topic :-PROBABILITY

1 (b)

Total number of ways placing 3 letters in three envelopes

$$= 3! = 3 \times 2 \times 1 = 6$$

Out of these ways only one way is correct

$$\therefore \text{The required probability} = \frac{1}{6}$$

2 (c)

Given probability of speaking truth are

$$P(A) = \frac{4}{5} \text{ and } P(B) = \frac{3}{4}$$

And their corresponds probabilities of not speaking truth are

$$P(\bar{A}) = \frac{1}{5} \text{ and } P(\bar{B}) = \frac{1}{4}$$

The probability that they contradict each other

$$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4}$$

$$= \frac{1}{5} + \frac{3}{20}$$

$$= \frac{7}{20}$$

3 (d)

Consider the following events:

A = Numbers on two tickets are not more than 10

B = Lowest number on two tickets is 5

$$\therefore \text{Required probability} = P(B/A)$$

$$\Rightarrow \text{Required Probability} = \frac{P(A \cap B)}{P(A)} = \frac{{}^5C_1 \times {}^1C_1 / {}^{100}C_2}{{}^{10}C_2 / {}^{100}C_2} = \frac{1}{9}$$

4 (b)

Let A_i denote an event of getting number i ($i = 1, 2, \dots, 6$) on each die. Then, A_i , $i = 1, 2, \dots, 6$ are mutually exclusive events

$$\therefore \text{Required probability} = P(A_1) + P(A_2) + \dots + P(A_6) \quad \dots(i)$$

Now,

$P(A_i)$ = Probability of getting number i on each die

$$\Rightarrow P(A_i) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

So, required probability = $\frac{6}{216} = \frac{1}{36}$ [From (i)]

ALITER We have,

Total number of elementary events = $6 \times 6 \times 6 = 216$

Same number can be obtained on each die in one of the following ways:

(1,1,1),(2,2,2),(3,3,3),(4,4,4),(5,5,5),(6,6,6)

Favourable number of elementary events = 6

Hence, required probability = $\frac{6}{216} = \frac{1}{36}$

5 **(b)**

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{7}}$$

$$= \frac{\frac{2}{15}}{\frac{1}{6} + \frac{2}{15} + \frac{1}{14}} = \frac{14}{39}$$

6 **(d)**

Total number of elementary events associated to the random experiment is $36 \times 36 = 36^2$

Throws of the two persons are equal means the sum of the numbers are same. The sum of the numbers can be 2,3,...,12.

∴ Favourable number of elementary events

$$= 2(1 \times 1 + 2 \times 2 + \dots + 3 \times 3 + 5 \times 5) + 6 \times 6 = 146$$

Hence, required probability = $1 - \frac{146}{36^2} = \frac{572}{648}$

7 **(c)**

At least one spade and one ace can be drawn in two mutually exclusive ways:

(i) Drawing one spade and one ace from 3 aces other than ace of spade

(ii) Drawing ace of spade and one other spade.

∴ Required probability

$$= \frac{{}^{13}C_1 \times {}^3C_1 + {}^{12}C_1 \times {}^1C_1}{{}^{52}C_2} = \frac{51}{{}^{52}C_2} = \frac{1}{26}$$

8 **(d)**

$$P(\text{selecting a black ball}) = \frac{{}^5C_1}{{}^{12}C_1}$$

$$P(\text{selecting a red ball}) = \frac{{}^3C_1}{{}^{12}C_1}$$

$$P(\text{black ball or red ball}) = \frac{{}^5C_1 + {}^3C_1}{{}^{12}C_1} = \frac{2}{3}$$

9 (b)

Given total number of bolts=600

Numbers of large bolts=20% of 600

$$= \frac{20}{100} \times 600 = 120$$

Number of small bolts=10% of 600

$$= \frac{10}{100} \times 600 = 60$$

∴ Number of suitable bolts

$$= 600 - 120 - 60 = 420$$

∴ Probability of selecting suitable bolt

$$= \frac{420}{600} = \frac{7}{10}$$

10 (d)

Let A denote the event that the student is selected in I.I.T. entrance test and B denotes the event that he is selected in Roorkee entrance test. Then,

$$P(A) = 0.2, P(B) = 0.5 \text{ and } P(A \cap B) = 0.3$$

$$\therefore \text{Required probability} = P(\bar{A} \cap \bar{B})$$

$$\Rightarrow \text{Required probability} = 1 - P(A \cup B)$$

$$\Rightarrow \text{Required probability} = 1 - \{P(A) + P(B) - P(A \cap B)\}$$

$$\Rightarrow \text{Required probability} = 1 - (0.2 + 0.5 - 0.3) = 0.6$$

11 (c)

The probability that A get r heads in the three tosses of a coin is

$P(X = r) = {}^3C_r \left(\frac{1}{2}\right)^3$. The probability that A and B both get r heads in three tosses of a coin is

$${}^3C_r \left(\frac{1}{2}\right)^3 \cdot {}^3C_r \left(\frac{1}{2}\right)^3$$

$$= ({}^3C_r)^2 \left(\frac{1}{2}\right)^6$$

$$\therefore \text{Required probability} = \sum_{r=0}^3 ({}^3C_r)^2 \left(\frac{1}{2}\right)^6$$

$$= \left(\frac{1}{2}\right)^6 (1 + 9 + 9 + 1) = \frac{20}{64} = \frac{5}{16}$$

12 (b)

We have, $P(S) = P\{5, 6\} = \frac{2}{6} = \frac{1}{3}$

Let us denote the occurrence of a number greater than 4 in a single throw of the die and F denote its failure.

$$\Rightarrow P(F) = \frac{2}{3}$$

P (an even number of tosses is needed)

$$= P(FS \text{ or } FFFS \text{ or } FFFFFS \text{ or } \dots)$$

$$= P(F)P(S) + P(F)^3P(S) + P(F)^5P(S) + \dots$$

$$= \frac{P(F)P(S)}{1 - P(F)^2} = \frac{\frac{2}{9}}{1 - \frac{4}{9}} = \frac{2}{5}$$

13 (c)

Let E = event when each American man is seated adjacent to his wife

and A = event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife.

Again, $n(E) = (5!) \times (2!)^4$

$$\therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)}$$

$$= \frac{(4!) \times (2!)^3}{(5!) \times (2!)^4} = \frac{2}{5}$$



14 (a)

Required probability = $\frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$

15 (b)

Let A_1 be the event that the black card is lost, A_2 be the event that red card is lost and let E be the event that first 13 cards examined are red. Then,

Required probability = $P(A_1/E)$

We have,

$P(A_1) = P(A_2) = 1/2$, as black and red cards were initially equal in number

Also, $P(E/A_1) = \frac{{}^{26}C_{13}}{{}^{51}C_{13}}$ and $P(E/A_2) = \frac{{}^{25}C_{13}}{{}^{51}C_{13}}$

\therefore Required probability = $P(A_1/E)$

$$= \frac{P(E/A_1)P(A_1)}{P(E/A_1)P(A_1) + P(E/A_2)P(A_2)} = \frac{\frac{1}{2} \times \frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2} \times \frac{{}^{26}C_{13}}{{}^{51}C_{13}} + \frac{1}{2} \times \frac{{}^{25}C_{13}}{{}^{51}C_{13}}} = \frac{2}{3}$$

16 (b)

The probability of getting at least one head in n tosses of a coin = $1 - \left(\frac{1}{2}\right)^n$

$\therefore 1 - \left(\frac{1}{2}\right)^n \geq 0.9 \Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1 \Rightarrow 2^n \geq 10 \Rightarrow n \geq 4$

Hence, the least value of n is 4

17 (b)

$$\text{Favourable ways} = 3! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \right) = 2$$

$$\text{Total ways} = 3!$$

$$\therefore \text{Probability} = \frac{2}{3!} = \frac{1}{3}$$

18 (a)

$$\therefore f(x) = x^3 + ax^2 + bx + c$$

$$\therefore f'(x) = 3x^2 + 2ax + b$$

$y = f(x)$ is increasing.

$$\Rightarrow f'(x) \geq 0, \forall x$$

And for $f'(x) = 0$ should not form an interval.

$$\Rightarrow 4a^2 - 4 \times 3 \times b \leq 0$$

$$\Rightarrow a^2 - 3b \leq 0$$

This is true for exactly 16 ordered pairs (a, b) , $1 \leq a, b \leq 6$ namely $(1, 1), (1, 2), (1, 3), (1, 4); (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6); (3, 3), (3, 4), (3, 5), (3, 6)$ and $(4, 6)$.

$$\text{Thus, required probability} = \frac{16}{36} = \frac{4}{9}$$

19 (b)

$$P(0 < x < 3) = P(x = 1) + P(x = 2)$$

$$= \frac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4} + \frac{{}^3C_2 \times {}^7C_2}{{}^{10}C_4} = \frac{3 \times 35}{210} + \frac{3 \times 21}{210} = \frac{4}{5}$$

20 (c)

$$\text{Given, } np = 4, npq = 2$$

$$\Rightarrow p = q = \frac{1}{2}$$

$$\therefore n = 4 \times 2 = 8$$

$$\therefore P(X > 6) = {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right) + {}^8C_8 \left(\frac{1}{2}\right)^8$$

$$= \frac{8}{256} + \frac{1}{256} = \frac{9}{256}$$

PE

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	D	B	B	D	C	D	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	B	B	B	A	B	C

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