

Topic :-PROBABILITY

1 (b)

Total number of ways = $6 \times 6 \times 6$

Favourable number of ways = 6

$$\therefore \text{Required probability} = \frac{6}{6 \times 6 \times 6} = \frac{1}{36}$$

2 (a)

Given, $P(A) = P(B) = x$

$$\text{and } P(A \cap B) = P(A' \cap B') = \frac{1}{3}$$

$$\therefore P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3} \quad \dots(i)$$

$$\text{Also, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{2}{3} = 2x - \frac{1}{3} \quad [\text{from Eq (i)}]$$

$$\Rightarrow x = \frac{1}{2}$$

3 (a)

$$S = \{00, 01, 02, \dots, 49\}$$

Let A be the event that sum of the digits on the selected ticket is 8, then

$$A = \{08, 17, 26, 35, 44\}$$

Let B be the event that the product of the digits is zero

$$B = \{00, 01, 02, 03, \dots, 09, 10, 20, 30, 40\}$$

$$\therefore A \cap B = \{8\}$$

$$\therefore \text{Required probability} = P\left(\frac{A}{B}\right)$$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$$

4 **(a)**

If any number the last digits can be 0,1,2,3,4,5,6,7,8,9. Therefore, last digit of each number can be chosen in 10 ways.

∴ The last digit of all numbers can be chosen in 10^n ways. If the last digit is to be 1,3,7, or 9, then none of the numbers can be even or end in 0 or 5. Thus, we have a choice of 4 digits viz. 1,3,7, or 9 with which each of n numbers should end.

So, favourable number of ways = 4^n

Hence, required probability = $\frac{4^n}{10^n} = \left(\frac{2}{5}\right)^n$

5 **(c)**

Consider the following events:

A = A worker receives bonus, B = A worker is skilled.

We have,

$$P(A) = \frac{30}{100} \text{ and } P(B/A) = \frac{20}{100}$$

∴ Required probability = $P(A \cap B) = P(A)P(B/A)$

$$\Rightarrow \text{Required probability} = \frac{30}{100} \times \frac{20}{100} = 0.06$$

6 **(d)**

∴ Required probability = $\frac{{}^{10}C_1 + {}^6C_1}{{}^{16}C_1}$

$$= \frac{16}{16} = 1$$

7 **(c)**

Let E = Event of getting a head from a coin

F = Event of getting an odd number {1, 3,5}, from a die

$$P(E) = \frac{1}{2}, P(F) = \frac{3}{6} = \frac{1}{2}$$

Since, E and F are independent events

$$\therefore P(E \cap F) = P(E) \cap P(F) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

8 **(a)**

Probability that at least one shot hits the plane

$$= 1 - P(\text{none of the shot hits the plane})$$

$$= 1 - 0.6 \times 0.7 \times 0.8 \times 0.9$$

$$= 1 - 0.3024 = 0.6976$$

9 **(c)**

Number of favourable cases (HTH, HTH) = 2

Number of total cases = $2^3 = 8$

$$\therefore \text{Required probability} = \frac{2}{8} = \frac{1}{4}$$

10 **(c)**

Consider the following events:

E_1 = Selecting first bag

E_2 = Selecting second bag

A = Getting a ticket bearing number 4

$$\therefore \text{Required probability} = P((E_1 \cap A) \cup (E_2 \cap A))$$

$$= P(E_1 \cap A) + P(E_2 \cap A)$$

$$= P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

$$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6} = \frac{5}{24}$$

11 **(b)**

We have,

$$\text{Required probability} = {}^6C_4\left(\frac{1}{2}\right)^6 + {}^6C_5\left(\frac{1}{2}\right)^6 + {}^6C_6\left(\frac{1}{2}\right)^6 = \frac{11}{32}$$

12 **(c)**

Let X denote the number of aces.

Probability of selecting aces,

$$P = \frac{4}{52} = \frac{1}{13}$$

Probability of not selecting aces,

$$q = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X = 1) = 2 \times \left(\frac{1}{13}\right) \times \left(\frac{12}{13}\right) = \frac{24}{169}$$

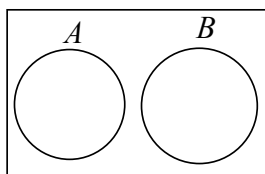
$$P(X = 2) = 2 \left(\frac{1}{13}\right)^2 \cdot \left(\frac{12}{13}\right)^0 = \frac{2}{169}$$

$$\text{Mean} = \sum P_1 X_i = \frac{24}{169} + \frac{2}{169} = \frac{2}{13}$$

13 **(d)**

$$P(A) = 0.45,$$

$$P(B) = 0.35 \quad (\text{events are mutually exclusive})$$



$$P(A \cap B) = 0$$

PE

14 (b)

Total cases = 4

Correct option = 1

So, probability of correct answer = $\frac{1}{4}$

15 (c)

$$P(E \cap F) = P(E) \cdot P(F)$$

$$\text{Now, } P(E \cap F) = P(E) - P(E \cap F) = P(E)[1 - P(F)]$$

$$= P(E) \cdot P(F^c)$$

$$\text{and } P(E^c \cap F^c) = 1 - P(E \cup F)$$

$$= 1 - [P(E) + P(F) - P(E \cap F)]$$

$$= [1 - P(E)][1 - P(F)] = P(E^c)P(F^c)$$

$$\text{Also } P(E/F) = P(E) \text{ and } P(E^c/F^c) = P(E^c)$$

$$\Rightarrow P(E/F) + P(E^c/F^c) = 1$$

16 (a)

One integer can be chosen out of 200 integers in ${}^{200}C_1$ ways. Let A be the event that an integer selected is divisible by 6 and B that it is divisible by 8

$$\text{Then, } P(A) = \frac{33}{200}, P(B) = \frac{25}{200}$$

$$\text{and } P(A \cap B) = \frac{8}{200}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}$$

17 (a)

Given, $np = 4, npq = 2$

$$\Rightarrow p = q = \frac{1}{2}, n = 8$$

We know, $P(X = r) = {}^nC_r p^r q^{n-r}$

$$\therefore P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 = 8 \times \frac{1}{2^8} = \frac{1}{32}$$

18 (b)

Let X be binomial variate with parameter $n = 100$ and P

Since, $P(X = 50) = P(X = 51)$ [given]

$$\Rightarrow {}^{100}C_{50}p^{50}(1-p)^{50} = {}^{100}C_{51}p^{51}(1-p)^{49}$$

$$\Rightarrow \frac{100!}{50!50!} \times \frac{51!49!}{100!} = \frac{p}{1-p}$$

$$\Rightarrow \frac{51}{50} = \frac{p}{1-p}$$

$$\Rightarrow p = \frac{51}{101}$$

19 (d)

Total number=90

Number divisible by 6 are {6,12,18,24,30,36,42,48,54,60,66,72,78,84,90}

Numbers divisible by 8 are {8,16,24,32,40,48,56,64,72,80,88}

Numbers divisible by 6 and 8 are {24,48,72}

Total number of numbers divisible by 6 or 8

$$= 15 + 11 - 3 = 23$$

$$\therefore \text{Required probability} = \frac{23}{90}$$

20 (a)

Let A_i denote the event that the number i appears on the die, and let E denote the event that only white balls are drawn. Then,

$$P(A_i) = \frac{1}{6} \text{ and } P(E/A_i) = \frac{{}^6C_i}{{}^{10}C_i}, i = 1, 2, \dots, 6$$

Required probability = $P(E)$

$$= P\left(\bigcup_{i=1}^6 (E \cap A_i)\right) = \sum_{i=1}^6 P(E \cap A_i) = \sum_{i=1}^6 P(A_i)P(E/A_i)$$

$$= \frac{1}{6} \left\{ \frac{6}{10} + \frac{15}{45} + \frac{20}{120} + \frac{15}{210} + \frac{6}{252} + \frac{1}{210} \right\} = \frac{1}{5}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	A	A	C	D	C	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	B	C	A	A	B	D	A

PE