

Topic :-PROBABILITY

1 (a)

The required probability is given by

$$\begin{aligned} & P\{(A \cap \bar{B}) \cup (\bar{A} \cap B)\} \\ &= P(A \cap \bar{B}) + P(\bar{A} \cap B) \quad \left[\begin{array}{l} \text{By add. Theorem for mutually} \\ \text{exclusive events} \end{array} \right] \\ &= P(A)P(\bar{B}) + P(\bar{A})P(B) \quad [\because A, B \text{ are independent events}] \\ &= P(A)(1 - P(B)) + (1 - P(A))P(B) \\ &= P(A) + P(B) - 2P(A)P(B) \\ &= P(A) + P(B) - 2P(A \cap B) \end{aligned}$$

2 (c)

Since, $P(A \cap B) = P(A)P(B)$

$\Rightarrow A$ and B are independent events

$\Rightarrow A^c$ and B^c will also independent events

Hence, $P(A \cup B)^c = P(A^c \cap B^c)$

$$= P(A^c)P(B^c)$$

3 (a)

Sum of Probabilities=1

$$\Rightarrow p + 2p + 3p + 4p + 5p + 7p + 8p + 9p$$

$$+ 10p + 11p + 12p = 1$$

$$\Rightarrow 72p = 1 \Rightarrow p = \frac{1}{72}$$

4 (b)

Let the total number of students be 100, then in which 60 girls and 40 boys

$$\text{As 25\% of boys offer Mathematics} = \frac{25}{100} \times 40 = 10 \text{ boys}$$

$$\text{and 10\% of girls offer Mathematics} = \frac{10}{100} \times 60 = 6 \text{ girls}$$

\therefore Total number of students, whose offers Mathematics is 16

$$\therefore \text{Required probability} = \frac{6}{16} = \frac{3}{8}$$

5 **(a)**

Let $q = 1 - p$. Since, head appears first time in an even throw 2 or 4 or 6

$$\therefore \frac{2}{5} = qp + q^3p + q^5p + \dots$$

$$\therefore \frac{2}{5} = \frac{qp}{1 - q^2}$$

$$\Rightarrow \frac{2}{5} = \frac{(1 - p)p}{1 - (1 - p)^2}$$

$$\Rightarrow \frac{2}{5} = \frac{1 - p}{2 - p}$$

$$\Rightarrow 4 - 2p = 5 - 5p \Rightarrow p = \frac{1}{3}$$

6 **(a)**

We have,

$$\begin{aligned} P[(E_1 \cup E_2) \cap (\bar{E}_1) \cap (\bar{E}_2)] \\ &= P[(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)] \\ &= P[(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2})] = P(\phi) = 0 \leq 1/4 \end{aligned}$$

7 **(b)**

$$\text{Given, } P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow P(A)P(B) = \frac{1}{6} \dots(i)$$

$$\text{and } P(\bar{A} \cap \bar{B}) = \frac{1}{3}$$

$$\Rightarrow P(\bar{A})P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow \{1 - P(A)\}\{1 - P(B)\} = \frac{1}{3}$$

$$\Rightarrow 1 - \frac{1}{3} + P(A)P(B) = P(A) + P(B)$$

$$\Rightarrow \frac{2}{3} + \frac{1}{6} = P(A) + P(B) \quad [\text{from Eq.(i)}]$$

$$\Rightarrow P(A) + P(B) = \frac{5}{6} \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$$

$$\text{or } P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$$

PE

9 (a)

We know, total probability distribution is 1.

$$\therefore \frac{1}{10} + k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$$

$$\Rightarrow \frac{6}{10} + 4k = 1$$

$$\Rightarrow k = \frac{1}{10}$$

10 (d)

We have, $p = 3/4$ and $n = 5$

\therefore Required probability

$$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2 + {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) + {}^5C_5 \left(\frac{3}{4}\right)^5 = \frac{459}{512}$$

11 (c)

One red card one queen can be drawn in the following mutually exclusive ways:

(I) By drawing one red card out of 24 red cards (excluding 2 red queens) and one red queen out of 2 red queens. Let this event be A

(II) By drawing one red card out of 26 red cards (including 2 red queens) and one queen out of 2 black queens. Let B

\therefore Required probability $= P(A \cup B) = P(A) + P(B)$

$$= \frac{{}^{24}C_1 \times {}^2C_1}{{}^{52}C_2} + \frac{{}^{26}C_1 \times {}^2C_1}{{}^{52}C_2} = \frac{50}{663}$$

12 (a)

Given, $4P(A) = 6P(B) = 10P(A \cap B) = 1$

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$$

13 (c)

Given, $np = 4$, $npq = 3V \Rightarrow p = \frac{1}{4}, q = \frac{3}{4}$

Mode is an integer x such that

$$\Rightarrow 4 + \frac{1}{4} > x > 4 - \frac{3}{4}$$

$$\Rightarrow 3.25 < x < 4.25 \therefore x = 4$$

14 (c)

$$P\left(\frac{B}{A \cup B^c}\right) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$$

$$\begin{aligned}
&= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)} \\
&= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)} \\
&= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}
\end{aligned}$$

15 **(b)**

The number of ways in which either player can choose a number from 1 to 25 is 25, so the total number of ways a choosing numbers is $25 \times 25 = 625$. So, the probability that they will not win a prize in a single trial

$$= 1 - \frac{1}{25} = \frac{24}{25}$$

16 **(c)**

Let X be the number of defective bulbs in a sample of 5 bulbs.

Probability that a bulb is defective $= p = \frac{10}{100} = \frac{1}{10}$

Then, $P(X = r) = {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r}$

$$\therefore \text{Required probability} = P(X = 0) = {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$$

17 **(b)**

We know sum of probability distribution is 1

$$\therefore k + 2k + 3k + 2k + k = 1$$

$$\Rightarrow k = \frac{1}{9}$$

$$\therefore \text{Mean, } m = \sum_{i=1}^5 P_i x_i$$

$$= k(1) + 2k(2) + 3k(3) + 2k(4) + k(5)$$

$$= k(1 + 4 + 9 + 8 + 5) = \frac{1}{9} \times 27 = 3$$

$$\therefore (k, m) = \left(\frac{1}{9}, 3\right)$$

18 **(d)**

$$\therefore \text{Required probability} = \frac{30 + 5}{60} = \frac{7}{12}$$

19 **(b)**

Total number of ways = ${}^{11}C_5 = 462$

Number of ways in which 2 particular girls are included

$${}^9C_3 = 84$$

$$\therefore \text{Required probability} = \frac{84}{462} = \frac{2}{11}$$

20 **(b)**

Required probability = $1 - P(\text{all letters in right envelope})$

$$= 1 - \frac{1}{n!}$$

PE

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	B	A	A	B	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	C	B	C	B	D	B	B

PE