

## Topic :-PROBABILITY

1 (a)

$$P(A \cup B') = P(A) + P(B') - P(A)P(B')$$

$$\therefore 0.8 = 0.3 + P(B') - 0.3P(B')$$

$$\Rightarrow 0.5 = P(B')(0.7)$$

$$\Rightarrow P(B') = \frac{5}{7}$$

$$\therefore P(B) = 1 - \frac{5}{7} = \frac{2}{7}$$

2 (a)

$$\text{Required probability} = \frac{3}{6} = \frac{1}{2}$$

3 (c)

There are two equilateral triangles in a regular hexagon

$$\therefore \text{Required probability} = \frac{2}{20} = \frac{1}{10}$$

4 (c)

From the given condition it is clear that a particular person is always in a committee of 3 persons. It means we have to select 2 person out of 37 persons.

$$\therefore \text{Required probability} = \frac{{}^{37}C_2}{{}^{38}C_3}$$

5 (d)

We know a leap year is fallen within 4 yr, so its probability =  $\frac{25}{100} = \frac{1}{4}$ .

In a century the probability of 53rd Sunday in a leap year =  $\frac{1}{4} \times \frac{2}{7} = \frac{2}{28}$

Non-leap year in century = 75

Probability of selecting is non-leap year =  $\frac{75}{100} = \frac{3}{4}$

53rd Sunday in non-leap year =  $\frac{1}{7}$

Similarly, in a century probabilities of 53rd Sunday in a non-leap year

$$= \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$$

$$\therefore \text{Required probability} = \frac{2}{28} + \frac{3}{28} = \frac{5}{28}$$

6 (b)

There are two conditions arise.

(i) When first is an ace of heart and second one is non-ace of heart, the probability =  $\frac{1}{52} \times \frac{51}{51} = \frac{1}{52}$

(ii) When first is non-ace of heart and second one is an ace of heart, the probability =  $\frac{51}{52} \times \frac{1}{51} = \frac{1}{52}$

∴ Required probability =  $\frac{1}{52} + \frac{1}{52} = \frac{1}{26}$

7 (c)

We have,

Total number of binary operations on  $A = n^{n^2}$

Total number of commutative binary operations on  $A$

$$= n^{\frac{n(n+1)}{2}}$$

$$\therefore \text{Required probability} = \frac{n^{\frac{n(n+1)}{2}}}{n^{n^2}} = \frac{n^{n/2}}{n^{n^2/2}}$$

8 (a)

$$\text{Required probability} = \frac{{}^{12}C_1}{{}^{20}C_1} = \frac{3}{5}$$

9 (b)

$P(A') = 1 - P(A) = 0.8$ ,  $P(A' \cap B)$  will maximum, if  $B \subseteq A'$  in which case  $A' \cap B = B$ . So,

$$P(A' \cap B) = P(B) = 0.5$$

10 (a)

The total number of ways in which 4 tickets can be drawn 5 times =  $4^5 = 1024$

The number of ways of getting a sum of 23

$$= \text{Coeff. of } x^{23} \text{ in } (x^{00} + x^{01} + x^{10} + x^{11})^5$$

$$= \text{Coeff. of } x^{23} \text{ in } [(1+x)(1+x^{10})]^5$$

$$= \text{Coeff. of } x^{23} \text{ in } (1+x)^5(1+x^{10})^5$$

$$= \text{Coeff. of } x^{23} \text{ in } \{(1+5x+10x^2+10x^3+5x^4+x^5)(1+5x^{10}+10x^{20}+10x^{30}+\dots)\}$$

$$= 100$$

$$\text{Hence, required probability} = \frac{100}{1024} = \frac{25}{256}$$

11 (c)

$$\text{Required probability} = {}^6C_4 \left(\frac{1}{4}\right)^4 \left(\frac{5}{6}\right)^2 = \frac{125}{15552}$$

12 (c)

In  $3n$  consecutive natural numbers, either

(i)  $n$  numbers are of from  $3P$

(ii)  $n$  numbers are of from  $3P + 1$

(iii)  $n$  numbers are of from  $3P + 2$

Here favourable number of cases = Either we can select three numbers from any of the set or we can select one from each set

$$= {}^nC_3 + {}^nC_3 + {}^nC_3 + ({}^nC_1 \times {}^nC_1 \times {}^nC_1)$$

$$= 3 \left( \frac{n(n-1)(n-2)}{6} \right) + n^3$$

$$= \frac{n(n-1)(n-2)}{2} + n^3$$

Total number of selections =  ${}^{3n}C_3$

∴ Required probability

$$= \frac{\frac{n(n-1)(n-2)}{2} + n^3}{\frac{3n(3n-1)(3n-2)}{6}}$$

$$= \frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$$

13 (d)

A and B will agree in a certain statement if both speak truth or both tell a lie. We define following events

$$E_1 = A \text{ and } B \text{ both speak truth} \Rightarrow P(E_1) = xy$$

$$E_2 = A \text{ and } B \text{ both tell a lie} \Rightarrow P(E_2) = (1-x)(1-y)$$

$E = A$  and  $B$  agree in a certain statement

Clearly,  $P(E|E_1) = 1$  and  $P(E|E_2) = 1$

The required probability is  $P(E_1|E)$

Using Bayes' theorem

$$P(E_1|E) = \frac{P(E_1)P(E|E_1)}{P(E_1)P(E|E_1) + P(E_2)P(E|E_2)}$$

$$= \frac{xy \cdot 1}{xy \cdot 1 + (1-x)(1-y) \cdot 1} = \frac{xy}{1-x-y+2xy}$$

14 (c)

∴ Total number of ways = 5!

and favourable number of ways =  $2 \cdot 4!$

$$\text{Hence, required probability} = \frac{2 \cdot 4!}{5!} = \frac{2}{5}$$

15 (c)

Three dice can be thrown in  $6^3 = 216$  ways.

The same number can appear on three dice in the following ways : (1,1,1),(2,2,2),(3,3,3),(4,4,4), (5,5,5),(6,6,6)

∴ Favourable number of elementary events = 6

$$\text{Hence, required probability} = \frac{6}{216} = \frac{1}{36}$$

16 (c)

$$\text{Probability that both are of red colours} = \frac{{}^8C_2}{{}^{15}C_2} = \frac{4}{15}$$

And probability that both are of black colours

$$= \frac{{}^7C_2}{{}^{13}C_2} = \frac{3}{15}$$

∴ Probability that they are of same colour

$$= \frac{4}{15} + \frac{3}{15} = \frac{7}{15}$$

17 **(b)**

Consider the following events :

A = Getting 2 black balls and 4 white in first 6 draws

B = Getting a black ball in 7th draw

Required probability =  $P(A \cap B) = P(A)P(B/A)$

$$\Rightarrow \text{Required probability} = \frac{{}^3C_2 \times {}^{10}C_4}{{}^{13}C_6} \times \frac{1}{7} = \frac{15}{286}$$

18 **(a)**

$P(A) = 0.25, P(B) = 0.50, P(A \cap B) = 0.14$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.25 + 0.50 - 0.14 = 0.61$$

∴  $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$$= 1 - 0.61 = 0.39$$

19 **(a)**

Total number of cases = 9999

Favourable cases =  $10 \times 9 \times 8 \times 7 = 5040$

$$\therefore \text{Probability} = \frac{5040}{9999}$$

20 **(d)**

Given,  $x^2 + 4x + c = 0$

For real roots,  $D = b^2 - 4ac \geq 0$

$$= 16 - 4c \geq 0$$

⇒  $c = 1, 2, 3, 4$  will satisfy the above inequality

$$\therefore \text{Required probability} = \frac{4}{9}$$

PE

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	D	B	C	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	D	C	C	C	B	A	A	D

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