DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :10

Topic :-PROBABILITY

1 **(a)**

Clearly, X is a binomial variate with $p = 1/2$

$\therefore P(X = r) = {}^nC_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r} = {}^nC_r\left(\frac{1}{2}\right)^n$

It is given that $P(X = 4)$,$P(X = 5)$ and $P(X = 6)$ are in AP

$\therefore 2\,P(X = 5) = P(X = 4) + P(X = 6)$

$\Rightarrow 2\ {}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow 2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$

$\Rightarrow 2 = \frac{5}{n-4} + \frac{n-5}{6}$

$\Rightarrow n^2 - 21\,n + 98 = 0 \Rightarrow n = 7, 14$

2 **(b)**

Three letters can be placed in 3 envelopes in 3! ways, whereas there is only one way of placing them in their right envelopes. So. Probability that all the letters are placed in the right envelopes $= \frac{1}{3!}$

Hence, required probability $= 1 - \frac{1}{3!} = \frac{5}{6}$

3 **(b)**

We have,

Total number of mappings from A to $B = n^m$

Number of injective mappings from A to $B = {}^nC_m \times m\,!$

Hence, required probability $= \frac{{}^nC_m \times m\,!}{n^m} = \frac{n\,!}{(n-m)\,!\,n^m}$

4 **(c)**

Let x be the probability of success in each trial, then $(1 - x)$ will be the probability of failure in each trial.

Thus, probability of exactly successes in a series of three trials

$= P(\overline{E}_1\,E_2\,E_3 + E_1\overline{E}_2\,E_3 + E_1\,E_2\overline{E}_3)$

$= (1 - x)x \cdot x + x(1 - x)x + x \cdot x(1 - x)$

$= 3x^2(1 - x)$

and the probability of three success

$P(E_1E_2E_3) = x \cdot x \cdot x = x^3$

According to question,

$9x^3 - 3x^2(1-x)$

$\Rightarrow 3x = 1 - x$

$\Rightarrow 4x = 1$

$\Rightarrow x = \dfrac{1}{4}$

Hence, the probability of success in each trial is $\frac{1}{4}$.

5 **(d)**

Let $E = E =$ Events of numbers divisible by 2 and 3 [ie, divisible by 6]

$= (6, 12, \ldots, 96)$

$n(E) = 16$

$\therefore$ Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3}$

$$= \frac{\frac{16 \times 15 \times 14}{3 \times 2 \times 1}}{\frac{100 \times 99 \times 98}{3 \times 2 \times 1}} = \frac{4}{1155}$$

6 **(d)**

Probability of getting a Sunday in a week,

$p = \dfrac{1}{7}, q = \dfrac{6}{7}$

Required probability $= {}^5C_2\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^3 = \frac{10 \times 6^3}{7^5}$

7 **(c)**

Given that, $np = 12$ ...(i)

and $\sqrt{npq} = 2 \Rightarrow npq = 4$ ...(ii)

From Eqs. (i) and (ii), we get

$12 \times q = 4 \Rightarrow q = \dfrac{1}{3}$

and we know that,

$p + q = 1 \Rightarrow p = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

8 **(c)**

Total cases $= {}^{52}C_4$

Favourable cases $= \left({}^{13}C_1\right)^4$

So, probability $= \frac{\left({}^{13}C_1\right)^4}{{}^{52}C_4}$

$$= \frac{13 \times 13 \times 13 \times 13 \times 1 \times 2 \times 3 \times 4}{52 \times 51 \times 50 \times 49}$$

$$= \frac{2197}{20825}$$

9 **(b)**

Required probability

$P(A_1 \cap A_2' \cap A_3) + P(A_1' \cap A_2 \cap A_3)$

$= P(A_1)P(A_2')P(A_3) + P(A_1')P(A_2)P(A_3)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

10 **(c)**

The last digit of the product will be 1, 2, 3, 4, 5, 6, 7, 8 or 9 if and only if each of the n positive integers ends in any of these digits. Now the probability of an integer ending in 1, 2, 3, 4, 5, 6, 7, 8 or 9 is $\frac{8}{10}$.Therefore the probability that the last digit of the product of n integer in 1, 2, 3, 4, 5, 6, 7, 8 or 9 is $\left(\frac{4}{5}\right)^n$.The probability for an integer to end in 1, 3, 7 or 9 is$\frac{4}{10} = \frac{2}{5}$

Therefore the probability for the product of n positive integers to end in 1, 3, 7 or 9 is $\left(\frac{2}{5}\right)^n$

Hence the required probability $= \left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n = \frac{4^n - 2^n}{5^n}$

11 **(d)**

Required probability $= P(WBWB) + P(BWBW)$

$= \left(\frac{{}^5C_1 \times {}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^8C_1 \times {}^7C_1 \times {}^6C_1 \times {}^5C_1}\right) + \left(\frac{{}^5C_1 \times {}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^8C_1 \times {}^7C_1 \times {}^6C_1 \times {}^5C_1}\right)$

$= \frac{1}{14} + \frac{1}{14} = \frac{2}{14} = \frac{1}{7}$

12 **(a)**

Let X denotes the number of red balls. Here probability of getting red balls, $p = \frac{3}{7}$ and probability of getting red bills, $q = \frac{4}{7}$

1. $P_1(X = 0) = {}^3C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^3 = \frac{64}{(7)^3}$

2. $P_2(X = 1) = {}^3C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^2 = \frac{144}{(7)^3}$

3. $P_3\,(X = 2) = {}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^1 \frac{108}{(7)^3}$

4. $P_4\,(X = 3) = {}^3C_3\left(\frac{3}{7}\right)^3 = \frac{27}{(7)^2}$

$\therefore$ Variance $= \sum_{i=0}^{3} P_i x_i^2 - \left(\sum_{i=0}^{3} P_i x_i\right)^2$

$= \left[\frac{64}{(7)^3} \times 0 + \frac{144}{(7)^3} \times (1)^2 + \frac{108}{(7)^3} \times (2)^2 + \frac{27}{(7)^3} \times (3)^2\right]$

$$-\left[\frac{64}{(7)^3}\times 0+\frac{144}{(7)^3}\times 1+\frac{108}{(7)^3}\times 2+\frac{27}{(7)^3}\times 3\right]^2$$

$$=\left[0+\frac{144}{343}+\frac{432}{343}+\frac{243}{343}\right]-\left[0+\frac{144}{343}+\frac{216}{343}+\frac{81}{343}\right]^2$$

$$=\frac{819}{343}-\left(\frac{441}{343}\right)^2$$

$$=\frac{280917-194481}{(343)^2}=\frac{36}{49}$$

Now, standard deviation $=\sqrt{\text{variance}}$

$$=\sqrt{\frac{36}{49}}=\frac{6}{7}$$

13 **(c)**

Since, $n(n+1)P=\frac{101}{3}$ is not an integer

Therefore, $P(X=r)$is maximum when $r=\left[\frac{101}{3}\right]=33$

14 **(c)**

We have,

$p=$ Probability that a bulb is defective $=\frac{5}{20}=\frac{1}{4}$

$\therefore q=1-p=1-\frac{1}{4}=\frac{3}{4}$

Let X denote the number of defective bulbs in a sample of 3 bulbs. Then, X is a binomial variate with parameter $n=3$ and $p=\frac{1}{4}$ such that

$P(X=r)=\ {}^3C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{3-r}$

$\Rightarrow P(X=2)=\ {}^3C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{2}\right)=\frac{9}{64}$

15 **(a)**

Given, $np=3=\sqrt{npq}=\frac{3}{2}$

$$\Rightarrow q=\frac{npq}{np}=\frac{9}{4\times 3}=\frac{3}{4}$$

$$\Rightarrow p=1-\frac{3}{4}=\frac{1}{4}$$

Also, $np=3\Rightarrow n=12$

Hence, binomial distribution is

$(q+p)^n=\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$

16 **(d)**

Since each element of a determinant of order 2 can be 0 or 1. Therefore, the total number of determinants with entries 0 or 1 is $2^4=16$. Out of these 16 determinants, there are 3 positive and 3 negative

$\therefore P(A)=P(B)=\frac{3}{16}\neq\frac{1}{2}$

17 **(a)**

$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$

$\therefore\ \frac{1}{15}=\frac{P(A\cap B)}{\frac{1}{12}}$

$\Rightarrow P(A\cap B)=\frac{1}{180}$

Also, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{1}{12}+\frac{5}{12}-\frac{1}{180}=\frac{89}{180}$

19 **(b)**

We have,

$np=20$ and $npq=4\Rightarrow q=\frac{1}{5}\Rightarrow p=\frac{4}{5}$

Now, $np=20\Rightarrow n=25$

20 **(d)**

Let Abe the event of obtaining an even sum and B be the event of obtaining a sum less five.

Then, we have to find $P(A\cup B)$.Since, A,B are not mutually exclusive, we have

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{18}{36}+\frac{6}{36}-\frac{4}{36}$

$=\frac{5}{9}$

[Since, there are 18 ways to get an even sum and 6 ways to get a sum less than 5 *ie*. (1,3), (3,1), (2,2), (1,2), (2,1),(1,1)

and 4 ways to get an even sum less than 5, *ie*, (1,3), (3,1), (2,2), (1,1).]

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	C	D	D	C	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	C	C	A	D	A	C	B	D