

## Topic :-PROBABILITY

1 (a)

Since the given distribution is a probability distribution

$$\begin{aligned} \therefore 0 + 2p + 2p + 3p + p^2 + 2p^2 + 7p^2 + 2p &= 1 \\ \Rightarrow 10p^2 + 9p - 1 &= 0 \Rightarrow (10p - 1)(p + 1) = 0 \Rightarrow p = 1/10 \end{aligned}$$

2 (a)

We have,

$$P(E \cap F) = \frac{1}{12} \text{ and } P(\bar{E} \cap \bar{F}) = \frac{1}{2}$$

$$\Rightarrow P(E)P(F) = \frac{1}{12} \text{ and } P(\bar{E})P(\bar{F}) = \frac{1}{2}$$

[  $\because$  E and F are independent events ]

$$\Rightarrow P(E)P(F) = \frac{1}{12} \text{ and } \{1 - P(E)\}\{1 - P(F)\} = \frac{1}{2}$$

$$\Rightarrow P(E)P(F) = \frac{1}{12} \text{ and } 1 - \{P(E) + P(F)\} + P(E)P(F) = \frac{1}{2}$$

$$\Rightarrow P(E)P(F) = \frac{1}{12} \text{ and } 1 - \{P(E) + P(F)\} + \frac{1}{12} = \frac{1}{2}$$

$$\Rightarrow P(E)P(F) = \frac{1}{12} \text{ and } P(E) + P(F) = \frac{7}{12}$$

The quadratic equation having  $P(E)$  and  $P(F)$  as its roots is

$$x^2 - \{P(E) + P(F)\}x + P(E)P(F) = 0$$

$$\Rightarrow x^2 - \frac{7}{12}x + \frac{1}{12} = 0 \Rightarrow x = \frac{1}{3}, \frac{1}{4}$$

$$\therefore P(E) = \frac{1}{3} \text{ and } P(F) = \frac{1}{4} \text{ or } P(E) = \frac{1}{4} \text{ and } P(F) = \frac{1}{3}$$

3 (a)

We have,

$$p = \text{Probability of getting at least 3 in a throw} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore q = 1 - p = \frac{1}{3}$$

Required probability

$$= {}^6C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^3 + {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^6C_6 \left(\frac{2}{3}\right)^6$$

$$= 41 \times \frac{2^4}{3^6}$$

4 (c)

Given  $P(A \cup B) = 0.6$ ,  $P(A \cap B) = 0.3$

$$\therefore P(A') + P(B')$$

$$= 1 - P(A) + 1 - P(B) = 2 - \{P(A) + P(B)\}$$

$$= 2 - \{P(A \cup B) + P(A \cap B)\}$$

$$= 2 - \{0.6 + 0.3\} = 2 - 0.9 = 1.1$$

6 (c)

Given,  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{3}$  and  $P(C) = \frac{1}{4}$

$$\therefore P(A') = \frac{2}{3}, P(B') = \frac{2}{3} \text{ and } P(C') = \frac{3}{4}$$

Now,  $P(A' \cap B' \cap C') = P(A')P(B')P(C')$

[ $\because A, B$  and  $C$  are independent events]

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{3}$$

7 (c)

Total number of elementary events =  $6^3 = 216$

Favourable number of elementary events

$$= \text{Coeff. of } x^{15} \text{ in } (x^1 + x^2 + x^3 + \dots + x^6)^3$$

$$= \text{Coeff. of } x^{15} \text{ in } x^3 \left( \frac{1 - x^6}{1 - x} \right)^3$$

$$= \text{Coeff. of } x^{12} \text{ in } (1 - 3x^6 + 3x^{12} - x^{18})(1 - x)^{-3}$$

$$= \text{Coeff. of } x^{12} \text{ in } (1 - x)^{-3} - 3 \text{ Coeff. of } x^6 \text{ in } (-x)^{-3}$$

$$+ 3 \text{ Coeff. of } x^0 \text{ in } (1 - x)^{-3}$$

$$= {}^{12+3-1}C_{3-1} - 3 \times {}^{6+3-1}C_{3-1} + 3 = {}^{14}C_2 - 3$$

$$= {}^{14}C_2 - 3 \times {}^8C_2 + 3 = 91 - 84 + 3 = 10$$

So, required probability =  $\frac{10}{216} = \frac{5}{108}$

8 (d)

For a Poisson distribution, mean = variance

$\Rightarrow$  Variance = 16

$$\therefore \text{Standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{16} = 4$$

9 (a)

The sum of two numbered on a dice is odd only, whence once is odd and second is even.

$\therefore$  Required probability

$$= 2 \times \text{probability of odd number}$$

$$\times \text{probability of even number}$$

[ $\because$  Here, we multiply by 2 because either the even number is on first or second dice.]

$$= 2 \times \left(\frac{5}{6}\right) \times \left(\frac{1}{6}\right) = \frac{5}{18}$$

10 (b)

In binomial distribution, variance =  $npq$  and mean =  $np$ .

From the given condition

$$npq = 3 \text{ and } np = 4$$

$$\therefore \frac{npq}{np} = \frac{3}{4}$$

$$\Rightarrow q = \frac{3}{4}, p = \frac{1}{4} \text{ and } n = 16$$

$$\text{Probability of exactly six success} = {}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$$

11 (a)

Since  $A$  and  $B$  are independent events

$$\therefore P(A \cap B) = \frac{1}{6} \text{ and } P(\bar{A} \cap \bar{B}) = \frac{1}{3}$$

$$\Rightarrow P(A)P(B) = \frac{1}{6} \text{ and } P(\bar{A})P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow P(A)P(B) = \frac{1}{6} \text{ and } \{(1 - P(A))\}\{(1 - P(B))\} = \frac{1}{3}$$

$$\Rightarrow 1 - [P(A) + P(B)] + \frac{1}{6} = \frac{1}{3}$$

$$\Rightarrow P(A) + P(B) = \frac{5}{6}$$

Solving  $P(A)P(B) = \frac{1}{6}$  and  $P(A) + P(B) = \frac{5}{6}$ , we get

$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{3} \text{ or } P(A) = \frac{1}{3} \text{ and } P(B) = \frac{1}{2}$$

Hence, option (a) is correct

12 (b)

Since,  $A$  and  $B$  are independent events

$$\therefore P(A)P(B) = \frac{1}{6} \text{ and } P(\bar{A})P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow [1 - P(A)][1 - P(B)] = \frac{1}{3}$$

$$\Rightarrow 1 - [P(A) + P(B)] + P(A)P(B) = \frac{1}{3}$$

$$\Rightarrow 1 + \frac{1}{6} - \frac{1}{3} = P(A) + P(B)$$

$$\Rightarrow P(A) + P(B) = \frac{5}{6}$$

$$\Rightarrow P(A) = \frac{1}{2}, P(B) = \frac{1}{3},$$

$$\text{or } P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$$

13 (a)

$$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, \dots$$

Therefore, for  $7^r$ ,  $r \in N$  the no. ends at unit place 7, 9, 3, 1, 7, ....

$\therefore 7^m + 7^n$  will be divisible by 5, if it ends at 5 or 0.

But it cannot end at 5.

Also it cannot end at 0.

For this  $m$  and  $n$  should be as follows :

	$m$	$n$
1	$4r$	$4r + 2$
2	$4r + 1$	$4r + 3$
3	$4r + 2$	$4r$
4	$4r + 3$	$4r + 1$

For any given value of  $m$ , there will be 25 values of  $n$ . Hence, the probability of the required event is  $\frac{100 \times 25}{100 \times 100} = \frac{1}{4}$ .

14 (c)

A dice is thrown thrice,  $n(S) = 6 \times 6 \times 6$

Favorable events of  $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

ie,  $(r_1, r_2, r_3)$  are ordered triplets which can take values,

$\{(1, 2, 3), (1, 5, 3), (4, 2, 3), (4, 5, 3)\}$   
 $\{(1, 2, 6), (1, 5, 6), (4, 2, 6), (4, 5, 6)\}$

ie, 8 ordered triplets and each can be arranged in  $3!$  ways = 6

$\therefore n(E) = 8 \times 6$

$$\Rightarrow P(E) = \frac{8 \times 6}{6 \times 6 \times 6}$$

$$= \frac{2}{9}$$

15 (c)

We have,

Total number of functions from  $A$  to itself =  $n^n$

Out of these functions,  $n!$  Function are injections

So, required probability =  $\frac{n!}{n^n} = \frac{(n-1)!}{n^{n-1}}$

16 (d)

Let  $A_i (i = 1, 2, 3, 4)$  be the event that the urn contains 2, 3, 4 or 5 white balls and  $E$  the event that two white balls are drawn. Since the four events  $A_1, A_2, A_3, A_4$  are equally likely. Therefore,  $P(A_i) = \frac{1}{4}, i = 1, 2, 3, 4$

We have,

$P(E/A_1)$  = Prob. that the urn contains 2 white balls and both have been drawn

$$\Rightarrow P(E/A_1) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$$

Similarly, we have

$$P(E/A_2) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10}, P(E/A_3) = \frac{{}^4C_2}{{}^5C_2} = \frac{3}{5}, P(E/A_4) = \frac{{}^5C_2}{{}^5C_2} = 1$$

$$\text{Required probability} = P(A_4/E) = \frac{P(A_4)P(E/A_4)}{\sum_{i=1}^4 P(A_i)P(E/A_i)}$$

$$= \frac{\frac{1}{4} \times 1}{\frac{1}{4} \left( \frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1 \right)} = \frac{1}{2}$$

17 **(b)**

There are 11 letters in word 'PROBABILITY' out of which 1 can be selected in  ${}^{11}C_1$  ways.

There are four vowels viz. A, I, O. Therefore,

Number of ways of selecting a vowel =  ${}^4C_1 = 4$

Hence, required probability =  $\frac{4}{11}$

18 **(b)**

If the show a six, then number of outcomes = 8

If die not show a six. Then number of outcomes = 2

$\therefore$  Sample space =  $1 \times 8 + 2 \times 5 = 18$  points

19 **(c)**

Given,  $n = 6$  and

$$P(X = 2) = 9P(X = 4)$$

$$\Rightarrow {}^6C_2 p^2 q^4 = 9 \cdot {}^6C_4 p^4 q^2$$

$$\Rightarrow 9p^2 = q^2$$

$$\Rightarrow p = \frac{1}{3}q$$

$\therefore$  We know that  $p + q = 1$

$$\Rightarrow \frac{q}{3} + q = 1$$

$$\Rightarrow q = \frac{3}{4} \text{ and } p = \frac{1}{4}$$

$\therefore$  Variance =  $npq$

$$= 6 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{9}{8}$$

20 **(a)**

$$\text{Required probability} = \frac{{}^5C_1 \times {}^8C_1}{{}^{13}C_2} + \frac{{}^5C_2}{{}^{13}C_2} = \frac{25}{39}$$

PE

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	C	A	C	C	D	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	C	C	D	B	B	C	A

PE