DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :8

Topic :-PERMUTATIONS AND COMBINATIONS

1 **(a)**

Total number of words

$=$ Number of arrangements of the letters of the word 'MATHEMATICS'

$= \dfrac{11\,!}{2\,!2\,!2\,!}$

2 **(c)**

We have,

$P_m = {}^mP_m = m\,!$

$\therefore 1 + P_1 + 2 \cdot P_2 + 3 \cdot P_3 + \ldots + n \cdot P_n$

$= 1 + 1 + 2 \cdot 2\,! + 3 \cdot 3\,! + \ldots + n \cdot n\,!$

$$= 1 + \sum_{r=1}^{n} r \cdot (r\,!) = 1 + \sum_{r=1}^{n} \{(r+1) - 1\} r\,!$$

$$= 1 + \sum_{r=1}^{n} [(r+1)\,! - r\,!]$$

$= 1 + \{(2\,! - 1\,!) + (3\,! - 2\,!) + (4\,! - 3\,!) + \ldots + ((n+1)\,! - n\,!)\}$

3 **(a)**

$\because \quad \dfrac{n(n-3)}{2} = 54$

$\Rightarrow \quad n^2 - 3n - 108 = 0$

$\Rightarrow \quad (n-12)(n+9) = 0$

$\Rightarrow \quad n = 12 \qquad [\because n \neq -9]$

4 **(c)**

In all, we have 8 squares in which 6 'X' have to be placed and It can be done in ${}^8C_6 = 28$ ways.

But this includes the possibility that either the top or horizontal row does not have any 'X'. Since, we want each row must have at least one 'X', these two possibilities are to be excluded.

Hence, required number of ways $= 28 - 2 = 26$

5 **(c)**

We have, 11 letters, viz. $A,A;I,I;N,N;E,X;M;T;O$

For groups of 4 we may arrange these as follows:

(i) Two alike, two others alike
(ii) Two alike, two different
(iii) all four different
(i) gives rise ${}^{3}C_2$ selections, (ii) gives rise $3 \times {}^{7}C_2$ selection and (iii) gives rise ${}^{8}C_4$ selections
So, the number of permutations

$$= 3\frac{4\,!}{2\,!2\,!} + 63\frac{4\,!}{2\,!} + 70.\,4\,! = 2454$$

6 **(b)**

There are total 20+1=21 persons. The two particular persons and the host be taken as one unit so that these remain $21 - 3 + 1 = 19$ persons be arranged in round table in 18! ways. But the two persons on either sides of the host can themselves be arranged in 2! ways.

$\therefore$ Required number of ways $= 2!18! = 2.18!$

7 **(d)**

${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$

$\Rightarrow \quad {}^{n}C_4 > {}^{n}C_3$

$\Rightarrow \quad \dfrac{n!}{(n-4)!4!} > \dfrac{n!}{(n-3)!3!}$

$\Rightarrow \;(n-3)(n-4)! > (n-4)!4$

$\Rightarrow \;n > 7$

8 **(c)**

We know that $\frac{(m\,n)!}{(m\,!)^n}$ is the number of ways of distributing mn distinct object in n persons equally

Hence, $\frac{(mn)\,!}{(m\,!)^n}$ is a positive integer

Consequently, $(mn)\,!$ is divisible by $(m\,!)^n$

Similarly, $(mn)\,!$ is divisible by $(n\,!)^m$

Now,

$n < m \Rightarrow m + n < 2m \leq mn$ and $m - n < m < mn$

$\Rightarrow (m+n)\,! | (mn)\,!$and $(m-n)\,! | (mn)\,!$

9 **(d)**

The number of subsets containing more than n elements is equal to

${}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \dots + {}^{2n+1}C_{2n+1}$

$= \dfrac{1}{2}\{ {}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_{2n+1}\} = \dfrac{1}{2}(2^{2n+1}) = 2^{2n}$

11 **(a)**

We have, 9 letters $3a$'s,$2b'$s and 4 c's. These 9 letters can be arranged in $\frac{9\,!}{3\,!\,2\,!\,4\,!} = 1260$ ways

12 **(c)**

The total number of subsets of given set is $2^9 = 512$

Case I When selecting only one even number {2, 4, 6, 8}

Number of ways = ${}^{4}C_1 = 4$

Case II When selecting only two even numbers = ${}^{4}C_2 = 6$

Case III When selecting only three even numbers = ${}^{4}C_3 = 4$

Case IV When selecting only four even numbers $= {}^4C_4 = 1$

$\therefore$ Required number of ways

$= 512 - (4 + 6 + 4 + 1) - 1 = 496$

[here, we subtract 1 for due to the null set]

14 **(a)**

The number of ways of choosing a committee if there is no restriction is

$${}^{10}C_4 \cdot {}^9C_5 = \frac{10\,!}{4\,!6\,!} \cdot \frac{9\,!}{4\,!5\,!} = 26460$$

The number of ways of choosing the committee if both Mr. A and Ms. B are included in the committee is ${}^9C_3 \cdot {}^8C_4 = 5880$

Therefore, the number of ways of choosing the committee when Mr. A and Ms. B are not together

$$= 26480 - 5880 = 20580$$

15 **(d)**

(1)It is true that product of r consecutive natural numbers is always divisible by r.

(2) Now, 115500=$2^2 \times 3^1 \times 5^3 \times 7^1 \times 11^1$

$\therefore$ Total number of proper divisor

$= (2 + 1)(1 + 1)(3 + 1)(1 + 1)(1 + 1) - 2$

$= 96 - 2 = 94$

(3) Total number of ways $= \frac{52!}{(13!)^4}$

Hence, all statements are true

16 **(d)**

Total numbers formed by using given 5 digits $= \frac{5!}{2!}$

For number greater than 40000, digit 2 cannot come at first place. Hence, number formed in which 2 is at the first place $= \frac{4!}{2!}$

Hence, total numbers formed greater than 40000

$= \frac{5!}{2!} = \frac{4!}{2!} = 60 - 12 = 48$

17 **(c)**

In the case of each book we may take 0,1,2,3,…p copies; that is, we may deal with each book in $p + 1$ ways and therefore with all the books in $(p + 1)^n$ ways. But, this includes the case where all the books are rejected and no selection is made

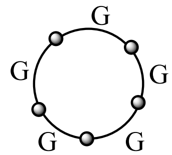
$\therefore$ Number of ways in which selection can be made

$= (p + 1)^n - 1$

18 **(b)**

First we fix the alternate position of the girls. Five girls can be seated around the circle in $(5 - 1)$ $! = 4!$, 5 boys can be seated in five vacant place by $5!$

$\therefore$ Required number of ways $= 4! \times 5!$



19 **(c)**

The number of words start with $D = 6! = 720$

The number of words start with $E = 6! = 720$

The number of words start with $MD = 5! = 120$

The number of words start with $ME = 5! = 120$

Now sthe first word start with MO is MODESTY.

Hence, rank of MODESTY $= 720 + 720 + 120 + 120$

$= 1681$

20 **(c)**

Starting with the letter A and arranging the other four letters, there are $4! = 24$ words. The starting with G, and arranging A, A, I, and N in different ways, there are $\frac{4!}{2!} = 12$ words. Next the 37^{th} word starts with I, there are 12 words starting with I. This accounts upto the 48^{th} word. The 49^{th} word in NAAGI. The 50^{th} word is NAAAIG

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	A	C	C	B	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	D	A	D	D	C	B	C	C