

**Topic :-PERMUTATIONS AND COMBINATIONS**

1 (c)

Since, no two lines are parallel and no three are concurrent, therefore  $n$  straight lines intersect at  ${}^n C_2 = N$  (say) points. Since, two points are required to determine a straight line, therefore the total number of lines obtained by joining  $N$  points is  ${}^N C_2$ . But in this each old line has been counted  $n-1$   ${}^n C_2$  times. Since, on each old line there will be  $n-1$  lines.

Hence, the required number of fresh lines.

$$\begin{aligned} &= {}^N C_2 - n {}^{n-1} C_2 \\ &= \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2} \\ &= \frac{{}^n C_2 ({}^n C_2 - 1)}{2} - \frac{n(n-1)(n-2)}{2} \quad (\because N = {}^n C_2) \\ &= \frac{\frac{n(n-1)}{2} \left( \frac{n(n-1)}{2} - 1 \right)}{2} - \frac{n(n-1)(n-2)}{2} \\ &= \frac{n(n-1)}{8} [(n^2 - n - 2) - 4(n-2)] \\ &= \frac{n(n-1)}{8} [n^2 - 5n + 6] \\ &= \frac{n(n-1)(n-2)(n-3)}{8} \end{aligned}$$

2 (c)

The required number of ways

$$\begin{aligned} &= ({}^2 C_1 \times {}^4 C_2 + {}^2 C_2 \times {}^4 C_1) \times 3! \\ &= (2 \times 6 + 1 \times 4)6 = 96 \end{aligned}$$

3 (a)

The factor of  $216 = 2^3 \cdot 3^3$

The odd divisors are the multiple of 3

$\therefore$  The number of divisors =  $3 + 1 = 4$

4 (c)

We have,

$$E_3(100!) = \left[ \frac{100}{3} \right] + \left[ \frac{100}{3^2} \right] + \left[ \frac{100}{3^3} \right] \left[ \frac{100}{3^4} \right] = 33 + 11 + 3 + 1 = 48$$

5 (b)

**Case I** When number in two digits.

Total number of ways =  $9 \times 9 = 81$

**Case II** When number in three digits

Total number of ways =  $9 \times 9 \times 9 = 729$

$\therefore$  Total number of ways =  $81 + 729 = 810$

6 (b)

We have,

$${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$$

$$\Rightarrow \frac{56!}{(50-r)!} = 3800 \left( \frac{54!}{(51-r)!} \right)$$

$$\Rightarrow 56 \times 55 = \frac{3800}{51-r}$$

$$\Rightarrow 51-r = 10 \Rightarrow r = 41$$

PE

7 (c)

We have,

Required number of numbers

= Total number of numbers formed by the digits 1,2,3,4,5

– Number of numbers having 1 at ten thousand's place

– Number of numbers having 2 at ten thousand's place and 1 at thousand's place

– Number of numbers having 2 at ten thousand's place and 3 at thousand's place

=  $5! - 4! - 3! - 3! = 120 - 24 - 6 - 6 = 84$

8 (c)

The number of ways of selecting 3 points out of 12 points is  ${}^{12}C_3$ . Three points out of 7 collinear points can be selected in  ${}^7C_3$  ways

Hence, the number of triangles formed =  ${}^{12}C_3 - {}^7C_3 = 185$

9 (d)

Required sum = (sum of the digits)  $(n-1)! \left( \frac{10^n - 1}{10 - 1} \right)$

$$= (1 + 2 + 3 + 4 + 5)(5-1)! \left( \frac{10^5 - 1}{10 - 1} \right)$$

$$= 360 \left( \frac{100000 - 1}{9} \right) = 40 \times 99999 = 3999960$$

10 (c)

Total number of words formed by 4 letters from given eight different letters with repetition =  $8^4$   
and number of words with no repetition =  ${}^8P_4$

$$\therefore \text{Required number of words} = 8^4 - {}^8P_4$$

11 (d)

Given number of flags = 5

Number of signals formed using one flag =  ${}^5P_1 = 5$

Similarly, using 2 flags =  ${}^5P_2$

Using 3 flags =  ${}^5P_3$

Using 4 flags =  ${}^5P_4$

Using 5 flags =  ${}^5P_5$

$\therefore$  Total number of signals that can be formed

$$= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$$

$$= 5 + 20 + 60 + 120 + 120$$

$$= 325$$

12 (d)

Required number of permutations =  $\frac{6!}{3!2!} = 60$

13 (c)

Out of 22 players 4 are excluded and 2 are to be included in every selection. This means that 9 players are to be selected from the remaining 16 players which can be done in  ${}^{16}C_9$  ways

234 (b)

The letters in the word 'CONSEQUENCE' are 2C, 3E, 2N, 1O, 1Q, 1S, 1U

$$\therefore \text{Required number of permutations} = \frac{9!}{2!2!}$$

15 (c)

The number of different sums of money Sita can form is equal to the number of ways in which she can select at least one coin out of 5 different coins

$$\text{Hence, required number of ways} = 2^5 - 1 = 31$$

16 (a)

For the first player, distribute the cards in  ${}^{52}C_{17}$  ways. Now, out of 35 cards left 17 cards can be put for second player in  ${}^{35}C_{17}$  ways. Similarly, for third player put them in  ${}^{18}C_{17}$  ways. One card for the last player can be put in  ${}^1C_1$  way. Therefore, the required number of ways for the proper distribution

$$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$$

$$= \frac{52!}{35!17!} \times \frac{35!}{18!17!} \times \frac{18!}{17!1!} \times 1! = \frac{52!}{(17!)^3}$$

17 (b)

Suppose  $x_1 x_2 x_3 x_4 x_5 x_6 x_7$  represents a seven digit number. Then  $x_1$  takes the value 1,2,3,...,9 and  $x_2, x_3, \dots, x_7$  all take values 0,1,2,3,...,9

If we keep  $x_1, x_2, \dots, x_6$  fixed, then the sum  $x_1 + x_2 + \dots + x_6$  is either even or odd. Since  $x_7$  takes 10 values 0,1,2,...,9, five of the numbers so formed will be even and 5 odd

Hence, the required number of numbers

$$= 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 4500000$$

19 **(a)**

The required number of ways is equal to the number of dearrangements of 10 objects.

$\therefore$  Required number of ways

$$= 10! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{1}{10!} \right\}$$

20 **(a)**

The number of words starting from A are  $=5!=120$

The number of words starting from I are  $=5!=120$

The number of words starting from KA are  $=4!=24$

The number of words starting from KI are  $=4!=24$

The number of words starting from KN are  $=4!=24$

The number of words starting from KRA are  $=3!=6$

The number of words starting from KRIA are  $=2!=2$

The number of words starting from KRIN are  $=2!=2$

The number of words starting from KRISA are  $=1!=1$

The number of words starting from KRISNA are  $=1!=1$

Hence, rank of the word KRISNA

$$= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324$$

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	C	A	C	B	B	C	C	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	D	C	B	C	A	B	D	A	A

PE