

## Topic :-PERMUTATIONS AND COMBINATIONS

1 (b)

The numbers formed will be divisible by 4 if the number formed by the two digits on the extreme right is divisible by 4, i.e. it should be 04,12,20,24,32,40

The number of numbers ending in 04 =  $3! = 6$

The number of numbers ending in 12 =  $3! - 2! = 4$

The number of numbers ending in 20 =  $3! = 6$

The number of numbers ending in 24 =  $3! - 2! = 4$

The number of numbers ending in 32 =  $3! - 2! = 4$

The number of numbers ending in 40 =  $3! = 6$

So, the required number =  $6 + 4 + 6 + 4 + 4 + 6 = 30$

2 (c)

The four girls can first be arranged in  $4!$  ways among themselves. In each of these arrangements there are 5 gaps (including the extremes) among the girls. Since the boys and girls are to alternate, we have to leave the first gap or last gap blank while arranging the boys. But, in each case the boys and girls can be arranged in  $4! \cdot 4!$  ways

$\therefore$  Required number of ways =  $2(4! \times 4!) = 2(4!)^2$

3 (a)

The product of  $r$  consecutive natural numbers

$$= 1.2.3.4.....r = r!$$

The natural number will be divided by  $r!$

4 (d)

The number of ways in which at least 5 women can be included in a committee

$$= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$$

(i) Women are in majority, then number of ways

$$= {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$$

$$= 2016 + 630 + 56 = 2702$$

(ii) Men are in majority, then number of ways

$$= {}^9C_5 \times {}^8C_7 = 126 \times 8 = 1008$$

5 (c)

In the number which is divisible by 5 and lying between 3000 and 4000, 3 must be at thousand place and 5 must be at unit place. Therefore rest of the digits (1, 2, 3, 4, 6) fill in two places. The number of ways =  ${}^4P_2$

6 (b)

We have 12 letters including 2 C's. Let us ignore 2 C's and thus we have 10 letters (4 A's, 3 B's, 1 D, 1 E, 1 F) and these 10 letters can be arranged in  $\frac{10!}{4!3!}$  ways.

Now, after arranging these 10 letters there will be 11 gaps in which two different letters can be arranged in  ${}^{11}P_2$  ways. But, since 2 C's are alike, the number of arrangements will be  $\frac{1}{2!} {}^{11}P_2 = \frac{11!}{9!2!}$

So, total number of ways in which C's are separated from one another =  $\frac{10!}{4!3!} \cdot \frac{11!}{9!2!} = 1386000$

7 (d)

An odd number has an odd digit at unit's place

So, unit's place can be filled in 4 ways

Each of ten's and hundred's place can be filled in 6 ways

Thousand place can be filled in 5 ways

Hence, required number of numbers =  $5 \times 6 \times 6 \times 4 = 720$

8 (c)

$$\therefore {}^{12}P_r = 1320 = 12 \times 11 \times 10$$

$$\Rightarrow \frac{12!}{(12-r)!} = 12 \times 11 \times 10$$

$$\therefore r = 3$$

9 (c)

Total time required = (total number of dials required to open the lock)  $\times$  5s

$$= 10^5 \times 5s$$

$$= \frac{500000}{60 \times 60 \times 13} \text{ days} = 10.7 \text{ days}$$

Hence, 11 days are enough to open the safe.

10 (a)

There are 6 rings and 4 fingers.

Since, each ring can be worn on any finger.

$$\therefore \text{Required number of ways} = 4^6$$

11 (c)

Consider the product

$$(x^0 + x^1 + x^2 \dots + x^9)(x^0 + x^1 + x^2 \dots + x^6) \dots 6 \text{ factors}$$

The number of ways in which the sum of the digits will be equal to 12 is equal to the coefficient of  $x^{12}$  in the above product. So, required number of ways

$$\begin{aligned}
 &= \text{Coeff. Of } x^{12} \text{ in } \left(\frac{1-x^{10}}{1-x}\right)^6 \\
 &= \text{Coeff. Of } x^{12} \text{ in } (1-x^{10})^6(1-x)^{-6} \\
 &= \text{Coeff. Of } x^{12} \text{ in } (1-x)^{-6}(1-{}^6C_1x^{10}+\dots) \\
 &= \text{Coeff. Of } x^{12} \text{ in } (1-x)^{-6} - {}^6C_1 \cdot \text{Coeff. of } x^2 \text{ in } (1-x)^{-6} \\
 &= {}^{12+6-1}C_{6-1} - {}^6C_1 \times {}^{2+6-1}C_{6-1} = {}^{17}C_5 - 6 \times {}^7C_5 = 6062
 \end{aligned}$$

12 (c)

We observe that a point is obtained between the lines of two of points on first line are joined by line segments to two points on the second line

Hence, required number of points =  ${}^nC_2 \times {}^nC_2$

14 (c)

Let there be 'n' men participants. Then, the number of games that the men play between themselves is  ${}^nC_2$  and the number of games that the men played with the women is  $2 \cdot (2n)$

$$\therefore 2 \cdot {}^nC_2 - 2 \cdot 2n = 66 \quad (\text{given})$$

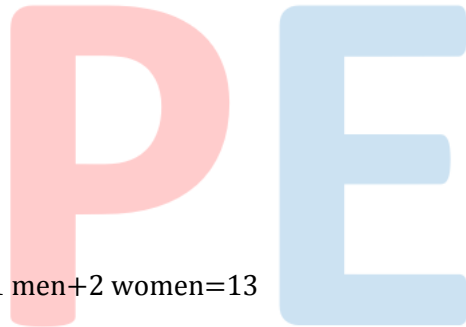
$$\Rightarrow n(n-1) - 4n - 66 = 0$$

$$\Rightarrow n^2 - 5n - 66 = 0$$

$$\Rightarrow (n+5)(n-11) = 0$$

$$\Rightarrow n = 11$$

$$\therefore \text{Number of participants} = 11 \text{ men} + 2 \text{ women} = 13$$



15 (b)

There are total  $20 + 1 = 21$  persons. The two particular persons and the host be taken as one unit so that these remaining  $21 - 3 + 1 = 19$  persons be arranged in round table in  $18!$  ways. But the two persons on either side of the host can themselves be arranged in  $2!$  ways

$$\therefore \text{required number of ways} = 2! \times 18!$$

16 (b)

Let the total number of persons in the room =  $n$

$$\therefore \text{Total number of handshakes} = {}^nC_2 = 66 \quad (\text{given})$$

$$\Rightarrow \frac{n!}{2!(n-2)!} = 66 \Rightarrow \frac{n(n-1)}{2} = 66$$

$$\Rightarrow n^2 - n - 132 = 0$$

$$\Rightarrow (n-12)(n+11) = 0$$

$$\Rightarrow n = 12 \quad [\because n \neq -11]$$

17 (d)

Given word is 'PENCIL'.

Total alphabets in the given word = 6

Number of vowels = 2 and number of consonants = 4

$$\therefore 4 \text{ consonants can be arranged in } 4! \text{ ways.}$$

∴ Remaining two places can be filled by two vowels in  ${}^5P_2$  ways.

∴ Total number of ways  $4! \times {}^5P_2 = 24 \times 20 = 480$

20     **(b)**

Let there are  $n$  teams.

Each team play to every other team in  ${}^nC_2$  ways

∴  ${}^nC_2 = 153$  (given)

$$\Rightarrow \frac{n!}{(n-2)!2!} = 153$$

$$\Rightarrow n(n-1) = 306$$

$$\Rightarrow n^2 - n - 306 = 0$$

$$\Rightarrow (n-18)(n+17) = 0$$

$$\Rightarrow n = 18 \quad (\because n \text{ is never negative})$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	A	D	C	B	D	C	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	C	B	B	D	C	B	B

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