

Topic :-PERMUTATIONS AND COMBINATIONS

1 (b)

Word MEDITERRANEAN has 2A, 3E, 1D, 1I, 1M, 2N, 2R, 1T

In out of four letters E and R is fixed and rest of the two letters can be chosen in following ways

Case I Both letter are of same kind ie, 3C_2 ways, therefore number of words = ${}^3C_2 \times \frac{2!}{2!} = 3$

Case II Both letters are of different kinds ie, 8C_2 ways, therefore number of words = ${}^8C_2 \times 2! = 56$

Hence, total number of words = $56 + 3 = 59$

2 (c)

Required number of ways

$$= \text{coefficient of } x^{2m} \text{ in } (x^0 + x^1 + \dots + x^m)^4$$

$$= \text{coefficient of } x^{2m} \text{ in } \left(\frac{1-x^{m+1}}{1-x}\right)^4$$

$$= \text{coefficient of } x^{2m} \text{ in } (1 - 4x^{m+1} + 6x^{2m+2} + \dots)(1-x)^{-4}$$

$$= {}^{2m+3}C_{2m} - 4^{m+2}C_{m-1}$$

$$= \frac{(2m+1)(2m+2)(2m+3)}{6} - \frac{4m(m+1)(m+2)}{6}$$

$$= \frac{(m+1)(2m^2+4m+3)}{3}$$

3 (c)

The number of times he will go to the garden is same as the number of selecting 3 children from 8.

Therefore, the required number of ways = ${}^8C_3 = 56$

4 (c)

The number of ways that the candidate may select

(i) if 2 questions from A and 4 question from B

$$= {}^5C_2 \times {}^5C_4 = 50$$

(ii) 3 question from A and 3 questions from B

$$= {}^5C_3 \times {}^5C_3 = 100$$

and (iii) 4 questions from A and 2 questions from B

$$= {}^5C_4 \times {}^5C_2 = 50$$

Hence, total number of ways = $50 + 100 + 50 = 200$

5 **(a)**

Since, $240 = 2^4 \cdot 3 \cdot 5$

$$\therefore \text{Total number of divisors} = (4 + 1)(1 + 1)(1 + 1) = 20$$

Out of these 2, 6, 10 and 30 are of the form $4n + 2$

7 **(a)**

Required number of arrangements

$$= \frac{6!}{2!3!} - \frac{5!}{3!} = 60 - 20 = 40$$

8 **(b)**

As we know the last two digits of $10!$ and above factorials will be zero-zero

$$\therefore 1! + 4! + 7! + 10! + 12! + 13! + 15! + 16! + 17!$$

$$= 1 + 24 + 5040 + 10! + 12! + 13! + 15! + 16! + 17!$$

$= 5065 + 10! + 12! + 13! + 15! + 16! + 17!$ in this series, the digit in the ten palce is 6 which is divisible by 3!

9 **(c)**

As the players who are to receive the cards are different

$$\text{So, the required number of ways} = \frac{52!}{(13!)^4}$$

10 **(c)**

We have, in all 12 points. Since 3 points are used to form a triangle, therefore the total number of triangles, including the triangles formed by collinear points on AB , BC and CA , is ${}^{12}C_3 = 220$. But, this includes the following:

The number of triangles formed by 3 points on AB

$$= {}^3C_3 = 1,$$

The number of triangles formed by 4 points on BC

$$= {}^4C_3 = 4,$$

The number of triangles formed by 5 points on CA

$$= {}^5C_3 = 10,$$

Hence, required number of triangles = $220 - (10 + 4 + 1) = 205$

11 **(b)**

Given, ${}^nP_r = 3024$

$$\Rightarrow \frac{n!}{(n-r)!} = 3024$$

And ${}^n C_r = \frac{n!}{r!(n-r)!}$

$$\Rightarrow 126 = \frac{3024}{r!}$$

$$\Rightarrow r! = 24 = 4!$$

$$\Rightarrow r = 4$$

12 **(d)**

We have,

$$\begin{aligned} & {}^{35}C_8 + \sum_{r=1}^7 {}^{42-r}C_7 + \sum_{s=1}^5 {}^{47-s}C_{40-s} \\ &= {}^{35}C_8 + \{ {}^{41}C_7 + {}^{40}C_7 + {}^{39}C_7 + {}^{38}C_7 + \dots + {}^{35}C_7 \} \\ &+ \{ {}^{46}C_{39} + {}^{45}C_{38} + \dots + {}^{42}C_{35} \} \\ &= {}^{35}C_8 + \{ {}^{35}C_7 + {}^{36}C_7 + \dots + {}^{41}C_7 \} \\ &+ \{ {}^{42}C_7 + {}^{43}C_7 + \dots + {}^{46}C_7 \} \quad [\because {}^n C_r = {}^n C_{n-r}] \\ &= ({}^{35}C_8 + {}^{35}C_7) + ({}^{36}C_7 + \dots + {}^{41}C_7 + \dots + {}^{46}C_7) \\ &= ({}^{36}C_8 + {}^{36}C_7) + {}^{37}C_7 + \dots + {}^{46}C_7 \\ &= {}^{37}C_8 + {}^{37}C_7 + {}^{38}C_7 + \dots + {}^{46}C_7 \\ &= \dots \\ &= {}^{46}C_8 + {}^{46}C_7 = {}^{47}C_8 \end{aligned}$$

13 **(b)**

Taking A_1, A_2 as one group we have 9 candidates which can be ranked in $9!$ ways. But A_1 and A_2 can be arranged among themselves in $2!$ ways

Hence, the required number = $(9!)(2!) = 2(9!)$

14 **(c)**

Considering AU as one letter, we have 4 letters, namely L, AU, G, H which can be permuted in $4!$ ways. But, A and U can be put together in $2!$ Ways.

Thus, the required number of arrangements = $4! \times 2! = 48$

155 **(c)**

Total number of ways in which all letters can be arranged in $6!$ ways.

There are two vowels in the word GARDEN

Total number of ways in which these two vowels can be arranged = $2!$

$$\therefore \text{Total number of required ways} = \frac{6!}{2!} = 360$$

16 **(a)**

The possible cases are

Case I A man invites 3 ladies and woman invites 3 gentleman

$$\Rightarrow {}^4 C_3 \cdot {}^4 C_3 = 16$$

Case II A man invites (2 ladies, 1 gentlemen) and woman invites (2 gentlemen, 1 lady)

$$\Rightarrow ({}^4 C_2 \cdot {}^3 C_1) \cdot ({}^3 C_1 \cdot {}^4 C_2) = 324$$

Case III A man invites (1 lady, 2 gentlemen) and woman invites (2 ladies, 1 gentlemen)

$$\Rightarrow ({}^4 C_1 \cdot {}^3 C_2) \cdot ({}^3 C_2 \cdot {}^4 C_1) = 144$$

Case IV A man invites (3 gentlemen) and woman invites (3 ladies)

$$\Rightarrow {}^3C_3 \cdot {}^3C_3 = 1$$

\therefore Total number of ways

$$= 16 + 324 + 144 + 1 = 485$$

18 **(a)**

A number between 5000 and 10,000 can have any of the digits 5,6,7,8,9 at thousand's place. So, thousand's place can be filled in 5 ways. Remaining 3 places can be filled by the remaining 8 digits in 8P_3 ways

Hence, required number = $5 \times {}^8P_3$

20 **(d)**

Two circles intersect maximum at two distinct points. Now, two circles can be selected in 6C_2 ways.

\therefore Total number of points in intersection are

$${}^6C_2 \times 2 = 30$$

The image shows the letters 'P' and 'E' in a large, bold, sans-serif font. The letter 'P' is colored in a light pink shade, and the letter 'E' is colored in a light blue shade. They are positioned side-by-side in the center of the page.

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	C	A	A	A	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	C	A	A	A	A	D

PE