

Topic :-PERMUTATIONS AND COMBINATIONS

1 (b)

$$\begin{aligned} \text{Now, } {}^n C_{r+1} + {}^n C_{r-1} + 2 \cdot {}^n C_r \\ = {}^n C_{r+1} + {}^n C_r + {}^n C_{r-1} + {}^n C_r \\ = {}^{n+1} C_{r+1} + {}^{n+1} C_r = {}^{n+2} C_{r+1} \end{aligned}$$

2 (a)

$$\begin{aligned} \frac{2}{9!} + \frac{2}{3!7!} + \frac{1}{5!5!} \\ = \frac{1}{1!9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{3!7!} + \frac{1}{9!1!} \\ = \frac{1}{10!} \left[\frac{10!}{1!9!} + \frac{10!}{3!7!} + \frac{10!}{5!5!} + \frac{10!}{3!7!} + \frac{10!}{9!1!} \right] \\ = \frac{1}{10!} \{ {}^{10} C_1 + {}^{10} C_3 + {}^{10} C_5 + {}^{10} C_7 + {}^{10} C_9 \} \\ = \frac{1}{10!} (2^{10} - 1) = \frac{2^9}{10!} = \frac{2^a}{b!} \text{ (given)} \end{aligned}$$

$$\Rightarrow a = 9, b = 10$$

3 (c)

Total number of lines obtained by joining 8 vertices of octagon is ${}^8 C_2 = 28$. Out of these, 8 lines are sides and remaining diagonal.

$$\text{So, number of diagonals} = 28 - 8 = 20$$

4 (b)

The number of times he will go to the garden is same as the number of selecting 3 children from 8 children

$$\therefore \text{The required number of times} = {}^8 C_3 = 56$$

5 (c)

$$\begin{aligned} \therefore {}^n C_r + {}^n C_{r-1} &= {}^{n+1} C_r \\ \therefore {}^{189} C_{36} + {}^{189} C_{35} &= {}^{190} C_{36} \\ \text{But } {}^{189} C_{35} + {}^{189} C_x &= {}^{190} C_x \end{aligned}$$

Hence, value of x is 36

6 (d)

$$\text{Required number of ways} = {}^3C_n = \frac{3n!}{n!2n!}$$

7 (a)

The word EXAMINATION has 2A, 2I, 2N, E, M, O, T, X therefore 4 letters can be chosen in following ways

Case I When 2 alike of one kind and 2 alike of second kind is 3C_2

$$\therefore \text{Number of words} = {}^3C_2 \times \frac{4!}{2!2!} = 18$$

Case II When 2 alike of one kind and 2 different *ie*, ${}^3C_1 \times {}^7C_2$

$$\therefore \text{Number of words} = {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$$

Case III When all are different *ie*, 8C_4

Hence, total number of words

$$= 18 + 756 + 1680 = 2454$$

8 (a)

$$\text{Required number of ways} = 5! \times 6!$$

9 (d)

Number of diagonals in a polygon of n sides

$$= {}^nC_2 - n$$

Here, $n = 20$

$$\therefore \text{required number of diagonals} = {}^{20}C_2 - 20$$

$$= \frac{20 \times 19}{2 \times 1} - 20 = 170$$

10 (c)

$${}^{47}C_4 + \sum_{r=1}^5 {}^{52-r}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4)$$

$$= {}^{52}C_4$$

11 (a)

First we fix the alternate position of 21 English book, in which 22 vacant places for Hindi books, hence total number of ways are ${}^{22}C_{19} = 1540$

12 (d)

Required number of ways

$$= \text{Total number of ways in which 8 boys can sit}$$

$$- \text{Number of ways in which two brothers sit together}$$

$$= 8! - 7! \times 2! = 7! \times 6 = 30240$$

13 (c)

In forming even numbers, the position on the right can be filled with either 0 or 2. When 0 is filled, the remaining positions can be filled in $3!$ ways, and when 2 is filled, the position on the left can be filled in 2 ways (0 cannot be used) and the middle two positions in $2!$ ways (0 can be used)

So, the number of even numbers formed = $3! + 2(2!) = 0$

15 **(a)**

Let the number of participants at the beginning was n

$$\therefore \frac{n(n-1)}{2} = 117 - 12$$

$$\Rightarrow n(n-1) = 2 \times 105$$

$$\Rightarrow n^2 - n - 210 = 0$$

$$\Rightarrow (n-15)(n+14) = 0$$

$$\Rightarrow n = 15 \quad [\because n \neq -14]$$

16 **(a)**

The number will be even if last digit is either 2, 4, 6 or 8 *ie* the last digit can be filled in 4 ways and remaining two digits can be filled in 8P_2 ways. Hence, required number of number of three different digits = ${}^8P_2 \times 4 = 224$

17 **(b)**

We have, $= {}^{x+2}P_{x+2} = (x+2)!$,

$$\text{and } b = {}^xP_{11} = \frac{x!}{(x-11)!}$$

$$\text{and } c = {}^{x-11}P_{x-11} = (x-11)!$$

Now, $a = 182bc$

$$\therefore (x+2)! = 182 \cdot \frac{x!}{(x-11)!} (x-11)!$$

$$\Rightarrow (x+2)! = 182x!$$

$$\Rightarrow (x+2)(x+1) = 182$$

$$\Rightarrow x^2 + 3x - 180 = 0$$

$$\Rightarrow (x-12)(x+15) = 0$$

$$\Rightarrow x = 12, -15$$

\therefore Neglect the negative value of x .

$$\Rightarrow x = 12$$

18 **(c)**

Since, the books consisting of 5 Mathematics, 4 physics, and 2 chemistry can be put together of the same subject is $5!4!2!$ ways

But these subject books can be arranged itself in $3!$ ways

\therefore Required number of ways = $5!4!3!2!$



19 (a)

If the function is one-one, then select any three from the set B in 7C_3 ways i.e., 35 ways.

If the function is many-one, then there are two possibilities. All three corresponds to same element number of such functions = ${}^7C_1 = 7$ ways. Two corresponds to same element. Select any two from the set B . The larger one corresponds to the larger and the smaller one corresponds to the smaller the third may corresponds to any two. Number of such functions = ${}^7C_2 \times 2 = 42$

So, the required number of mappings = $35 + 7 + 42 = 84$

20 (b)

The number of ordered triples of positive integers which are solution of $x + y + z = 100$

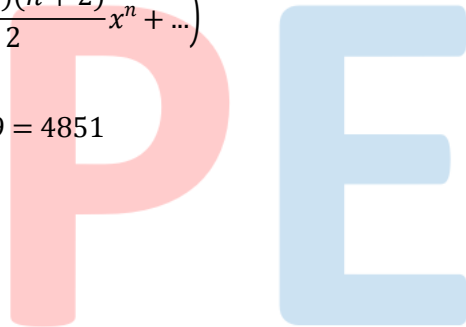
$$= \text{coefficient of } x^{100} \text{ in } (x + x^2 + x^3 + \dots)^3$$

$$= \text{coefficient of } x^{100} \text{ in } x^3(1 - x)^{-3}$$

$$= \text{coefficient of } x^{97} \text{ in}$$

$$\left(1 + 3x + 6x^2 + \dots + \frac{(n+1)(n+2)}{2}x^n + \dots\right)$$

$$= \frac{(97+1)(97+2)}{2} = 49 \times 99 = 4851$$



ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	B	C	D	A	A	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	B	A	A	B	C	A	B

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