

## Topic :-PERMUTATIONS AND COMBINATIONS

1 (b)

$$\because f(x_i) \neq y_i$$

ie, no object goes to its scheduled place. Then, number of one-one mappings

$$= 6! \left( 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} \right)$$

$$= 6! \left( \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} \right)$$

$$= 360 - 120 + 30 - 6 + 1 = 265$$

2 (d)

We have,

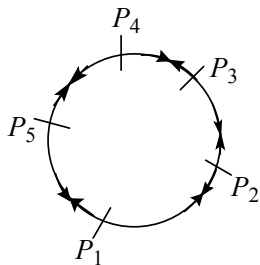
Required number of ways

$$= {}^{m+n}C_m \times (m-1)! \times (n-1)! = \frac{(m+n)!}{m n}$$

3 (a)

$\because$  Remaining 5 can be seated in  $4!$  ways.

Now, on cross marked five places 2 person can sit in  ${}^5P_2$  ways



So, number of arrangements

$$= 4! \times \frac{5!}{3!}$$

$$= 24 \times 20 = 480 \text{ ways}$$

4

**(a)**

$$\text{Given, } {}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$$

$$\Rightarrow \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)2n}{(n+2)(n+1)n} = \frac{3}{5}$$

$$\Rightarrow 10(2n+1) = 3(n^2 + 3n + 2)$$

$$\Rightarrow 3n^2 - 11n - 4 = 0$$

$$\Rightarrow (3n+1)(n-4) = 0$$

$$\Rightarrow n = 4 \quad \left( n \neq -\frac{1}{3} \right)$$

5

**(b)**

Required number of ways

$$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$= 60 + 120 + 60 + 6$$

$$= 246$$

6

**(a)**

$$\text{Required number of ways} = {}^8C_5$$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

The total number of ways a voter can vote

$$= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$$

$$= 8 + 28 + 56 + 70 + 56 = 218$$

7

**(c)**

From the first set, the number of ways of selection two lines =  ${}^4C_2$

From the second set, the number of ways of selection two lines =  ${}^3C_2$

Since, these sets are intersect, therefore they form a parallelogram,

$$\therefore \text{Required number of ways} = {}^4C_2 \times {}^3C_2$$

$$= 4 \times 3 = 12$$

8

**(b)**

Since, a set of  $m$  parallel lines intersecting a set of another  $n$  parallel lines in a plane, then the number of parallelograms formed is  ${}^mC_2 \times {}^nC_2$ .

9

**(a)**

$$\begin{aligned}
& {}^{50}C_4 + \sum_{r=1}^6 {}^{56-r}C_3 \\
&= {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3 \\
&= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 \\
&[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}] \\
&= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 \\
&= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 \\
&= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3 = {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4
\end{aligned}$$

10

**(a)**

Total number of four digit numbers =  $9 \times 10 \times 10 \times 10$

$$= 9000$$

Total number of four digit numbers which divisible by 5

$$= 9 \times 10 \times 10 \times 2 = 1800$$

$$\therefore \text{Required number of ways} = 9000 - 1800 = 7200$$

11

**(a)**

Man goes from Gwalior to Bhopal in 4 ways and they come back in 3 ways.

$$\therefore \text{Total number of ways} = 4 \times 3 = 12 \text{ ways}$$

12

**(c)**

Here, we have 1 M, 4 I's, 4 S's and 2 P's

$\therefore$  Total number of selections

$$= (1 + 1)(4 + 1)(2 + 1) - 1 = 149$$

13

**(c)**

Number of lines from 6 points =  ${}^6C_2 = 15$

Points of intersection obtained from these lines =  ${}^{15}C_2 = 105$

Now, we find the number of times, the original 6 points come.

Consider one point say  $A_1$ . Joining  $A_1$  to remaining 5 points, we get 5 lines and any two lines from these 5 lines gives  $A_1$  as the point of intersection.

$$\therefore A_1 \text{ is common in } {}^5C_2 = 10 \text{ times out of } 105 \text{ points of intersections.}$$

Similar is the case with other five points.

$$\therefore 6 \text{ original points come } 6 \times 10 = 60 \text{ times in points of intersection.}$$

Hence, the number of distinct points of intersection

$$= 105 - 60 + 6 = 51$$

15 **(b)**

At first we have to accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore, number of ways are  ${}^6P_5$  and rest of the five animals arrange in  $5!$  ways.

$$\text{Total number of ways} = 5! \times {}^6P_5$$

$$= 120 \times 720 = 86400$$

16 **(b)**

$$T_n = {}^nC_3 \quad \text{and} \quad T_{n+1} - T_n = 21$$

$$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$$

$$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$$

$$\Rightarrow {}^nC_2 = 21$$

$$\Rightarrow \frac{n(n-1)}{2} = 21$$

$$\Rightarrow n^2 - n - 42 = 0$$

$$\Rightarrow (n-7)(n+6) = 0$$

$$\therefore n = 7 \quad [\because \neq -6]$$

17 **(b)**

Total number of ways

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$$

$$= 10 + 45 + 120 + 210 = 385$$

18 **(b)**

The total number of two factors product =  ${}^{n+2}C_8$ . The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore, total number of two factors product, which are not multiple of 5, is  ${}^{160}C_2$

$$\text{Hence, required number of factors} = {}^{200}C_2 - {}^{160}C_2$$

$$= 19900 - 12720$$

$$= 7180$$

19 **(b)**

Total number of  $m$ -elements subsets of  $A = {}^nC_m \dots(i)$

and number of  $m$ -elements subsets of  $A$  each containing the element  $a_4 = {}^{n-1}C_{m-1}$

According to question,  ${}^nC_m = \lambda \cdot {}^{n-1}C_{m-1}$

$$\Rightarrow \frac{n}{m} \cdot {}^{n-1}C_{m-1} = \lambda \cdot {}^{n-1}C_{m-1}$$

$$\Rightarrow \lambda = \frac{n}{m} \quad \text{or} \quad n = m\lambda$$

20

**(a)**

The number of 1 digit numbers = 9

The number of 2 digit non-repeated numbers =  $9 \times 9 = 81$

The number of 3 digit non-repeated number

$$= 9 \times {}^9P_2 = 9 \times 9 \times 8 = 648$$

$$\therefore \text{Required number of ways} = 9 + 81 + 648 = 738$$

The image shows two large, stylized letters. The letter 'P' is on the left and is colored a light red or pink. The letter 'E' is on the right and is colored a light blue. Both letters have a rounded, sans-serif font style.

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	A	A	B	A	C	B	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	B	B	B	B	B	A

PE