DPP
DAILY PRACTICE PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :8

Topic :-LINEAR INEQUALITIES

1 **(c)**

We have,

$\log_{16} x^3 + (\log_2 \sqrt{x})^2 < 1$

$\Rightarrow \frac{3}{4}\log_2 x + \frac{1}{4}(\log_2 x)^2 < 1$

$\Rightarrow (\log_2 x)^2 + 3\log_2 x - 4 < 0$

$\Rightarrow (\log_2 x + 4)(\log_2 x - 1) < 0$

$\Rightarrow -4 < \log_2 x < 1 \Rightarrow 2^{-4} < x < 2 \Rightarrow x \in (1/16, 2)$

Also, LHS of the given inequality is defined fro $x > 0$

Hence, $x \in (1/16, 2)$

2 **(b)**

Since, $\sin x \le \cos^2 x$, becuase $\cos x$ must be a positive proper fraction

$\sin^2 x + \sin x - 1 \le 0$

Or $\left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \le 0$

From the definition of logarithm

$\sin x > 0,\ \cos x > 0,\ \cos x \ne 1$

$\therefore \sin x + \frac{1}{2} \le \frac{\sqrt{5}}{2},$

$\Rightarrow 0 < \sin x \le \frac{\sqrt{5}-1}{2}$

3 **(d)**

If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$

Then, $f(x)$ is minimum and $g(x)$ is maximum at

$f(x) = \frac{-D}{4a}, \quad \left(\because x = -\frac{-b}{a} \text{ and } f(x) = \frac{-D}{4a}\right)$

$\therefore \quad \min\{f(x)\} = \frac{-(4b^2 - 8c^2)}{4} = (2c^2 - b^2)$

And $\max\{g(x)\} = -\frac{(4c^2 + 4b^2)}{4(-1)} = (b^2 + c^2)$

Since, $\min f(x) > \max g(x) \Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow \quad c^2 > 2b^2 \quad \Rightarrow \quad |c| > \sqrt{2}|b|$

4 **(a)**

We have, $[x] + (x) = 5$

If $x \leq 2$, then $[x] + (x) \leq 2 + 2 < 5$

If $x \geq 3$, then $[x] + (x) \geq 3 + 3 > 5$

If $2 < x < 3$, then $[x] + (x) = 2 + 3 = 5$

Hence, the solution set is (2, 3)

5 **(a)**

We have,

$|x - x^2 - 1| = |2x - 3 - x^2|$

$\Rightarrow |x^2 - x + 1| = |x^2 - 2x + 3|$

$\Rightarrow x^2 - x + 1 = x^2 - 2x + 3 \quad \left[\begin{array}{c} \because x^2 - x + 1 > 0 \\ \text{and } x^2 - 2x + 3 > 0 \text{ for all } x \end{array}\right]$

$\Rightarrow x = 2$

6 **(a)**

Given, $\frac{|x-1|}{x+2} - 1 < 0$

Case I When $x < 1$, $|x - 1| = 1 - x$

$\therefore \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0$

$\Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2 \text{or} x > -\frac{1}{2}$

But $x < 1$

$\therefore x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right)$

Case II When $x \geq 1$, $|x - 1| = x - 1$

$\therefore \frac{x-1}{x+2} - 1 < 0 \Rightarrow -\frac{3}{x+2} < 0$

$\Rightarrow \frac{3}{x+2} > 0$

$\Rightarrow x > -2$

But $x \geq 1$

$\therefore x \geq 1$, *ie*, $x \in [1, \infty)$...(iii)

$\therefore$ From Eqs. (i) and (ii), we get

$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$

7 **(d)**

Given that, $\frac{x+2}{x^2+1} > \frac{1}{2}$

$\Rightarrow \quad x^2 - 2x - 3 < 0$

$\Rightarrow \quad (x - 3)(x + 1) < 0$

$\Rightarrow -1 < x < 3$

The integer value of x are 0, 1, 2

$\therefore$ The number of integral solutions are 3

8 **(b)**

$\because \tan\left(x+\frac{\pi}{3}\right) \geq 1 \Rightarrow \frac{\pi}{4} \leq x+\frac{\pi}{3} < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{12} \leq x < \frac{\pi}{6}$

$\Rightarrow n\pi - \frac{\pi}{12} \leq x \leq n\pi + \frac{\pi}{6}$

9 **(c)**

We have,

$\frac{1+(\log_a x)^2}{1+\log_a x} > 1$

$\Rightarrow \frac{1+(\log_a x)^2}{1+\log_a x} - 1 > 0$

$\Rightarrow \frac{(\log_a x)(\log_a x - 1)}{(1+\log_a x)} > 0$

$\Rightarrow -1 < \log_a x < 0$ or, $\log_a x > 1$

$\Rightarrow a^{-1} > x > a^0$ or, $x < a [\because 0 < a < 1]$

$\Rightarrow x \in (1, 1/a) \cup (0, a) [\because a > 0]$

10 **(b)**

We have,

$x = \frac{y+2}{y+1}$

$\Rightarrow y = \frac{2-x}{x-1}$

$\Rightarrow \left(\frac{2-x}{x-1}\right)^2 > 2 \quad [\because y^2 > 2]$

$\Rightarrow (2-x)^2 > 2(x-1)^2 \Rightarrow x^2 < 2$

11 **(a)**

Let $x = y - \frac{3\pi}{4}$. Then,

$\sin x = -\left(\frac{\cos y + \sin y}{\sqrt{2}}\right)$ and $\cos x = -\left(\frac{\cos y - \sin y}{\sqrt{2}}\right)$

$\Rightarrow \sin x + \cos x = -\sqrt{2}\cos y$ and $\sin x \cos x = \frac{1}{2}(2\cos^2 y - 1)$

Now,

$|\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$

$= |(\sin x + \cos x) + (\tan x + \cot x) + (\sec x + \operatorname{cosec} x)|$

$= \left|(\sin x + \cos x) + \frac{1}{\sin x \cos x} + \frac{\sin x + \cos x}{\sin x \cos x}\right|$

$$=\left|(\sin x+\cos x)\left(1+\frac{1}{\sin x\cos x}\right)+\frac{1}{\sin x\cos x}\right|$$

$$=\left|-\sqrt{2}\cos y\left(1+\frac{2}{2\cos^2 y-1}\right)+\frac{2}{2\cos^2 y-1}\right|$$

$$=\left|-\sqrt{2}\cos y-\frac{2(\sqrt{2}\cos y-1)}{2\cos^2 y-1}\right|$$

$$=\left|-\sqrt{2}\cos y-\frac{2}{\sqrt{2}\cos y+1}\right|=\left|\sqrt{2}\cos y+\frac{2}{\sqrt{2}\cos y+1}\right|$$

$$=\left|\lambda+\frac{2}{\lambda+1}\right|,\text{ where } \lambda=\sqrt{2}\cos y$$

$$=\left|(\lambda+1)+\frac{2}{\lambda+1}-1\right|\geq\left|(\lambda+1)+\frac{2}{(\lambda+1)}\right|-1$$

$$\geq 2\sqrt{(\lambda+1)\times\frac{2}{(\lambda+1)}}-1=2\sqrt{2}-1 \text{ [Using AM} \geq \text{GM]}$$

12 **(b)**

$$\frac{9\cdot 3^{2x}-6\cdot 3^x+4}{9\cdot 3^{2x}+6\cdot 3^x+4}=\frac{(3^{(x+1)})^2-2(3^{(x+1)})+4}{(3^{(x+1)})^2+2(3^{(x+1)})+4}$$

$$=\frac{t^2-2t+4}{t^2+2t+4} \quad (\text{where } t=3^{x+1}) \quad ...(i)$$

Since, $\frac{1}{3}<\frac{x^2-2x+4}{x^2+2x+4}<3$

$\therefore$ From Eq.(i), the given expression lies between 1/3 and 3

13 **(c)**

Using A.M. $\geq$ G.M., we have

$4^x+4^{1-x}\geq 2\sqrt{4^x\times 4^{1-x}}\Rightarrow 4^x+4^{1-x}\geq 4$

14 **(b)**

We have,

$3^{-|x|}-2^{|x|}=0\Rightarrow 3^{-|x|}=2^{|x|}\Rightarrow 6^{|x|}=1\Rightarrow x=0$

16 **(d)**

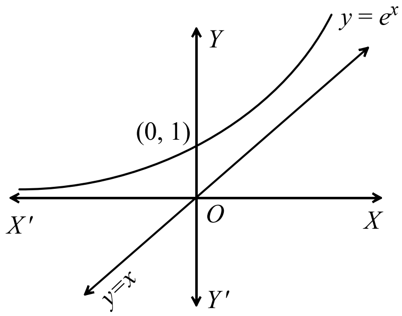
We have,

$P\leq(x_1+x_2+...+x_{n-1})(x_2+x_4+x_6+...+x_n)$

$\Rightarrow P\leq\frac{1}{4}(x_1+x_2+...+x_n)^2=\frac{1}{4}$

17 **(b)**

We observe that $y=e^{-x}$ and $y=x$ intersect at exactly one point. So, the equation $e^{-x}=x$ has exactly one real root:



18 **(b)**

We have,

$3^x + 3^{1-x} - 4 < 0$

$\Rightarrow (3^x)^2 - 4(3^x) + 3 < 0$

$\Rightarrow (3^x - 1)(3^x - 3) < 0$

$\Rightarrow 1 < 3^x < 3 \Rightarrow 0 < x < 1 \Rightarrow x \in (0, 1)$

20 **(c)**

We know that $x + \frac{1}{x} \geq 2$ for all $x > 0$

$\therefore \sin^5 \theta + \operatorname{cosec}^5 \theta \geq 2 \text{for} 0 < \theta < \pi$

Hence, the minimum value of $\sin^5 \theta + \operatorname{cosec}^5 \theta$ is 2

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	D	A	A	A	D	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	B	B	D	B	B	B	C