DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :7

Topic :-LINEAR INEQUALITIES

1 **(a)**

Using A.M. $\geq$ G.M., we have

$a+b \geq 2\sqrt{ab}, b+c \geq 2\sqrt{bc}$ and $c+a \geq 2\sqrt{ca}$

$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$

$\Rightarrow (2-a)(2-b)(2-c) \geq 8abc \left[\begin{array}{c} \because a+b+c=2 \\ \therefore b+c=2-a \text{ etc} \end{array}\right]$

2 **(a)**

We have, $x^2+y^2+z^2=27$

Now,

$$\frac{(x^2)^{3/2}+(y^2)^{3/2}+(z^2)^{3/2}}{3} \geq \left(\frac{x^2+y^2+z^2}{z}\right)^{3/2}$$

$\Rightarrow x^3+y^3+z^3 \geq 81$

3 **(c)**

Given, $2x-7<11, 3x+4<-5$

$\Rightarrow \ x<9, x<-3$

$\Rightarrow \ x<-3$

$\therefore x$ lies in the interval $(-\infty, -3)$

4 **(c)**

Let

$$f(x)=x^8-x^5-\frac{1}{x}+\frac{1}{x^4}=\frac{x^{12}-x^9-x^3+1}{x^4}=\frac{(x^9-1)(x^3-1)}{x^4}$$

Clearly, $f(x) \geq 0$ for all $x<0$ and it is not defined for $x=0$

For $0<x<1$, we have

$x^9-1<0$ and $x^3-1<0 \Rightarrow f(x)>0$

For $x \geq 1$, we have $x^9-1 \geq 0$ and $x^3-1 \geq 0 \Rightarrow f(x) \geq 0$

Hence, $f(x) \geq 0$ for all $x \neq 0$

5 **(c)**

We have,

$5^{(1/4)(\log_5 x)^2} \geq 5\, x^{(1/5)(\log_5 x)}$

$$\frac{1}{4}(\log_5 x)^2 \log_5 5 \geq \log_5 5+\frac{1}{5}(\log_5 x)\log_5 x$$

$\Rightarrow (\log_5 x)^2 \geq 20$

$\Rightarrow (\log_5 x)^2 - (2\sqrt{5})^2 \geq 0$

$\Rightarrow \log_5 x \leq -2\sqrt{5}$or, $\log_5 x \geq 2\sqrt{5}$

$\Rightarrow x \leq 5^{-2\sqrt{5}}$or, $x \geq 5^{2\sqrt{5}}$

$\Rightarrow x \in \left(0, 5^{-2\sqrt{5}}\right] \cup \left[5^{2\sqrt{5}}, \infty\right)$ [$\log_5 x$ is defined for $x > 0$]

6 **(a)**

We know that,

AM $\geq$ GM

$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow 4 \geq \sqrt{ab}$ $(\because a + b = 8\text{given})$

$\Rightarrow ab \leq 16$

Equality holds when number are equal. So, ab is equal to 16 when $a = 4, b = 4$

7 **(a)**

Curves $y = \cos x$ and $y = -|x|$ do not intersect. So, the equation $\cos x + |x| = 0$ has no real root

8 **(d)**

Using A.M. $\geq$ G.M., we have

$\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 2\sqrt{\frac{\cos^3 x}{\sin x} \times \frac{\sin^3 x}{\cos x}}$for all $x \in (0, \pi/2)$

$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq \sin 2x$ for all $x \in (0, \pi/2)$

$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 1$ for all $x \in (0, \pi/2)$

9 **(d)**

$x^2 + 4ax + 20$

$\therefore$ $(4a)^2 - 4 \times 2 < 0$

[$\because$ if $f(x) > 0$, then]

$\Rightarrow$ $16a^2 < 8 \Rightarrow a^2 < \frac{1}{2}$

$\Rightarrow$ $-\frac{1}{\sqrt{2}} < a < \frac{1}{\sqrt{2}}$

10 **(b)**

Using A.M. $\geq$ G.M., we have

$\frac{a+b}{2} > \sqrt{ab}$ $[\because a \neq b]$

$\Rightarrow a + b > 2\sqrt{ab}$

11 **(a)**

We have,

$x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$

Now, two cases arise

CASE I$When x \geq 3$:

In this case, we have $|x-3| = x-3$

So, the given equation reduces to

$x^2 \cdot 2^{x+1} + 2^{x-1} = x^2 \cdot 2^{x+1} + 2^{x-1}$

Which is an identity in x and hence it is true for all $x \geq 3$

CASE II$When x < 3$:

In this case, we have $|x-3| = -(x-3)$

So, the given equation reduces to

$x^2 \cdot 2^{x+1} + 2^{-x+5} = x^2 \cdot 2^{-x+7} + 2^{x-1}$

$\Rightarrow x^2 2^{x+1} - 2^{x-1} = x^2 \cdot 2^{-x+7} - 2^{-x+5}$

$\Rightarrow 2^{x-1}(4x^2 - 1) = 2^{-x+5}(4x^2 - 1)$

$\Rightarrow 2^{2x}(4x^2 - 1) = 2^6(4x^2 - 1)$

$\Rightarrow (2^{2x} - 2^6)(4x^2 - 1) = 0$

$\Rightarrow 2x = 6$or, $4x^2 - 1 = 0$

$\Rightarrow x = 3$ or, $x = \pm \frac{1}{2}$

But, $x < 3$. Therfore, $x = \pm \frac{1}{2}$

Hence, the given equation has no negative integral root

12 **(b)**

We have,

$2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 120$

$\Rightarrow 2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 2 \times 3 \times 4 \times 5$

$\Rightarrow \sin^2 x \log 2 + \cos^2 y \log_3 + \sin^2 z \log 4 + \cos^2 \omega \log 5 \geq \log 2 + \log 3 + \log 4 + \log 5$

$\Rightarrow \cos^2 x \log 2 + \sin^2 y \log 3 + \cos^2 z \log 4 + \sin^2 \omega \log 5 \leq 0$

$\Rightarrow \cos^2 x = 0, \sin^2 y = 0, \cos^2 z = 0$ and $\sin^2 \omega = 0$

$\Rightarrow x = m\pi \pm \frac{\pi}{2}, m \in Z; y = n\pi, n \in Z$

$z = r\pi \pm \frac{\pi}{2}, r \in Z;\ \omega = t\,\pi, t \in Z$

But, $x, y, z, \omega \in [0,10]$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, y = 0, \pi, 2\pi, 3\pi, z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

and $\omega = 0, \pi, 2\pi, 3\pi$

Hence, the number of ordered 4-tuples is $3 \times 4 \times 3 \times 4 = 144$

13 **(d)**

We have,

$\log_b a + \log_a b + \log_d c + \log_c d$

$= \left(\log_b a + \frac{1}{\log_b a}\right) + \left(\log_d c + \frac{1}{\log_d c}\right) \geq 2 + 2 = 4$

14 **(c)**

We have,

$\log_{1/3}(2^{x+2}-4^x) \geq -2$

$\Rightarrow 2^{x+2}-4^x \leq \left(\frac{1}{3}\right)^{-2}$ and $2^{x+2}-4^x > 0$

$\Rightarrow 4(2^x)-(2^x)^2 \leq 9$ and $2^x(2^2-2^x) > 0$

$\Rightarrow (2^x)^2-4(2^x)+9 \geq 0$ and $2^x < 2^2$

$\Rightarrow x < 2$ $[\because (2^x)^2-4(2^x)+9 > 0$ for all $x \in R]$

$\Rightarrow x \in (-\infty, 2)$

15 **(c)**

Given, $\frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$\Rightarrow \frac{-3x-2}{(x+1)(x+2)(2x+1)} > 0$

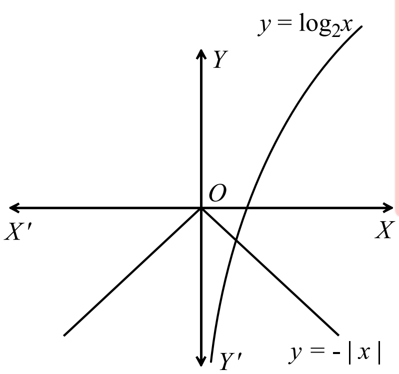
Equating each factor equal to 0, we have

$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$

It is clear $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$

16 **(b)**

We observe that the curves $y = \log_2 x$ and $y = -|x|$ intersect at exactly one point. So, the equation $\log_2 x + |x| = 0$ has exactly one real root



17 **(a)**

Using A.M. ≥ G.M., we have

$\frac{bcx+cay+abz}{3} \geq (bcx \times cay \times abz)^{1/3}$

$\Rightarrow bcx+cay+abz \geq 3(a^2b^2c^2 \times xyz)^{1/3}$

$\Rightarrow bcx+cay+abz \geq 3abc \quad [\because xyz = abc]$

18 **(a)**

We have,

$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14} \leq 4-2x-x^2$

$\Rightarrow \sqrt{3(x+1)^2+4}+\sqrt{5(x+1)^2+9} \leq (x+1)^2+5$

Clearly, LHS ≥ 5 and LHS ≤ 5

So, the inequation holds when each side is equal to 5

This is true when $x = -1$

Hence, the given inequation has exactly one solution

19 **(b)**

Let a_1,a_2,a_3,...,a_n be the lengths of n parts of the stick. Then,

$a_1 + a_2 + a_3 + ... + a_n = 20$ and $a_1 a_2 a_3 ... a_n > 1$

Now, A.M. ≥ G.M.

$\Rightarrow \frac{a_1 + a_2 + ... + a_n}{a_n} \geq (a_1 a_2 ... a_n)^{1/n}$

$\Rightarrow \frac{20}{n} > 1 \Rightarrow n < 20$

$\therefore$ Maximum possible value of n is 19

20 **(b)**

$\because$ AM $> GM$

$$\frac{(a + b - c) + (b + c - a)}{2} > \sqrt{(a + b - c)(b + c - a)}$$

$\Rightarrow b > \sqrt{(a + b - c)(b + c - a)}$...(i)

Similarly, $\frac{(b + c - a) + (c + a - b)}{2} > \sqrt{(b + c - a)(c + a - b)}$

$\Rightarrow c > \sqrt{(b + c - a)(c + a - b)}$...(ii)

and $\frac{(c + a - b) + (a + b - c)}{2} > \sqrt{(c + a - b)(a + b - c)}$

$\Rightarrow a > \sqrt{(c + a - b)(a + b - c)}$...(iii)

On multiplying relations (i), (ii) and (iii), we get

$abc > (a + b - c)(b + c - a)(c + a - b)$

$\Rightarrow (a + b - c)(b + c - a)(c + a - b) - abc < 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	C	A	A	D	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	D	C	C	B	A	A	B	B