CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :6

Topic :-LINEAR INEQUALITIES

1 **(d)**

The LHS of the given inequality is meaningful for $x > 0$ and $x \neq 1$

Now,

$\log_3 x - \log_x 27 < 2$

$\Rightarrow \log_3 x - 3\log_x 3 < 2$

$\Rightarrow \log_3 x - \dfrac{3}{\log_3 x} < 2$

$\Rightarrow \dfrac{(\log_3 x)^2 - 3 - 2(\log_3 x)}{\log_3 x} < 0$

$\Rightarrow \dfrac{(\log_3 x - 3)(\log_3 x + 1)}{(\log_3 x - 0)} < 0$

$\Rightarrow \log_3 x < -1$ or, $0 < \log_3 x < 3$

$\Rightarrow x < 3^{-1}$or, $3^0 < x < 3^3 \Rightarrow x < \dfrac{1}{3}$ or, $1 < x < 27$

Also, $x > 0$ and $x \neq 1$

$\therefore x \in (0,1/3) \cup (1, 27)$

2 **(c)**

Given inequation is $x^2 - 2x + 5 \leq 0$

$\therefore$ Roots are

$x = \dfrac{-2 \pm \sqrt{4-20}}{2} = \dfrac{2 \pm 4i}{2}$

$\therefore$ Roots are imaginary, therefore no real solutions exist

3 **(b)**

We have,

$\dfrac{2^{\sin^2 x} + 2^{\cos^2 x}}{2} \geq \sqrt{2^{\sin^2 x} \times 2^{\cos^2 x}}$ [Using A.M. ≥ G.M.]

$\Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2\sqrt{2} \Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2$

4 **(a)**

Given that, $a^2 + b^2 + c^2 = 1$ ...(i)

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow 2(ab+bc+ca)\geq -1$ [from Eq.(i)]

$\Rightarrow ab+bc+ca\geq -\frac{1}{2}$...(iii)

Also, $a^2+b^2+c^2-ab-bc-ca$

$$=\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}\geq 0$$

$\Rightarrow ab+bc+ca\leq a^2+b^2+c^2$

$\Rightarrow ab+bc+ca\leq 1$ [from Eq.(i)] ...(iii)

From relation (ii) and (iii), we get

$$-\frac{1}{2}\leq ab+bc+ca\leq 1$$

5 **(b)**

We have,

$\sqrt{4x+9}-\sqrt{11x+1}=\sqrt{7x+4}$

$\Rightarrow\sqrt{4x+9}-\sqrt{7x+4}=\sqrt{11x+1}$

$\Rightarrow 4x+9+7x+4-2\sqrt{(4x+9)(7x+4)}=11x+1$

$\Rightarrow -2\sqrt{(4x+9)(7x+4)}=-12$

$\Rightarrow(4x+9)(7x+4)=36$

$\Rightarrow 28x^2+79x=0\Rightarrow x=0,-\dfrac{79}{28}$

Clearly, only $x=0$ satisfies the given equation

6 **(a)**

Since, $-3<x+\frac{2}{x}<3$

$\Rightarrow -3<\frac{(x^2+2)x}{x^2}<3$ $\Rightarrow -3x^2<(x^2+2)x<3x^2 (x\neq 0)$

$\Rightarrow x(x^2+3x+2)>0$

And $x(x^2-3x+2)<0$ $(x\neq 0)$

$\Rightarrow x(x+1)(x+2)>0$

And $x(x-1)(x-2)<0$

$\Rightarrow x\in(-2,-1)\cup(0,\infty)$...(i)

And $x\in(-\infty,0)\cup(1,2)$...(ii)

From relations (i) and (ii), we get

$x\in(-2,-1)\cup(1,2)$

7 **(b)**

We have a,b,c are sides of a triangle

$\therefore b+c-a>0, c+a-b>0, a+b-c>0$

Let $x=b+c-a, y=c+a-b, z=a+b-c$

$\Rightarrow y+z=2a, z+x=2b, x+y=2c$

Now, $\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}$

$=\frac{y+z}{2x}+\frac{z+x}{2y}+\frac{x+y}{2z}$

$=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+\frac{x}{z}+\frac{y}{x}+\frac{z}{y}\right)$

$\geq\frac{6}{2}\left(\frac{y}{x}\cdot\frac{x}{y}\cdot\frac{y}{z}\cdot\frac{z}{y}\cdot\frac{z}{x}\cdot\frac{x}{z}\right)$ $(\because \text{AM} \geq \text{GM})$

$=3$

8 **(b)**

Let d_1,d_2 be the lengths of diagonals and θ be the angle between them. Then,

Area $=\frac{1}{2}d_1d_2\sin\theta \Rightarrow a^2=\frac{1}{2}d_1d_2\sin\theta \Rightarrow d_1d_2=\frac{2a^2}{\sin\theta}$

Using A.M. $\geq$ G.M., we have

$\frac{d_1+d_2}{2}\geq\sqrt{d_1d_2}\Rightarrow d_1+d_2\geq 2\sqrt{\frac{2a^2}{\sin\theta}}\geq 2\sqrt{2}a$

9 **(c)**

We have, $|2x-3|<|x+5|$

$\Rightarrow|2x-3|-|x+5|<0$

$$\Rightarrow\begin{cases} 3-2x+x+5<0, x\leq -5 \\ 3-2x-x-5<0, -5<x\leq\frac{3}{2} \\ 2x-3-x-5<0, x>\frac{3}{2}\end{cases}$$

$$\Rightarrow\begin{cases} x>8, x\leq -5 \\ x>-\frac{2}{3}, -5<x\leq\frac{3}{2} \\ x<8, x>\frac{3}{2}\end{cases}$$

$\Rightarrow x\in\left(-\frac{2}{3},\frac{3}{2}\right]\cup\left(\frac{3}{2},8\right)$

$\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

10 **(c)**

We have,

$|a_n| < 2$ i.e. $-2 < a_n < 2$

$\therefore \max\left(1 + a_1x + a_2x^2 + \ldots a_nx^n\right)$

$= 1 + 2|x| + 2|x|^2 + \ldots + 2|x|^n$

$= 1 + 2|x|\left\{\frac{1-|x|^n}{1-|x|}\right\}$

$= 1 + 2 \cdot \frac{1}{3}\left\{\frac{1-1/3^n}{1-1/3}\right\} > 0$

and,

$\min\left(1 + a_1x + a_2x^2 + \ldots + a_nx^n\right)$

$= 1 - 2|x| - 2|x|^2 \ldots - 2|x|^n$

$= -2[1 + |x| + |x|^2 + \ldots + |x|^n] + 3$

$= -2\left\{\frac{1-|x|^n}{1-|x|}\right\} + 3$

$= -2\left\{\frac{1-1/3^n}{1-1/3}\right\} + 3 > 0$

Thus, the curve $y = 1 + a_1x + a_2x^2 + \ldots + a_nx^n$ does not meet x-axis for $|x| < 1/3$ and $|a_n| < 2$

Hence, the equation has no real roots

12 **(c)**

Since, $|r| < 1 \Rightarrow -1 < r < 1$

Also, $a = 5(1-r)$

$$\Rightarrow 0 < a < 10\begin{bmatrix}\because \text{ at } r = -1, & a = 0 \\ \text{andat } r = 1, & a = 0\end{bmatrix}$$

13 **(a)**

Consider the curves $y = e^{x-8}$ and $y = 17 - 2x$.These two curves intersect at (8, 1) only. Hence, the equation $e^{x-8} + 2x - 17 = 0$ has exactly one root which is equal to 8

14 **(b)**

Let $x^2 + 18x + 30 = y$. Then,

$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$

$\Rightarrow y = 2\sqrt{y + 15}$

$\Rightarrow y^2 - 4y - 60 = 0 \Rightarrow (y-10)(y+6) = 0 \Rightarrow y = 10$

$\therefore x^2 + 18x + 30 = y \Rightarrow x^2 + 18x + 20 = 0$

$\therefore$ Product of roots $= 20$

15 **(a)**

Since, $2 \le |x-3| < 4$

$\Rightarrow 2 \le x - 3 < 4$

Or $2 \le -(x-3) < 4$

$\Rightarrow 5 \le x < 7$ or $-1 \le -x < 1$

$\Rightarrow\ 5 \le x < 7 \text{or} -1 < x \le 1$

$\therefore\ x \in (-1, 1] \cup [5, 7)$

16 **(b)**

Given that, $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$

$$\therefore\ x = \frac{a+2b}{a+b} = \frac{\frac{a}{b}+2}{1+\frac{a}{b}} = 1 + \frac{1}{\frac{a}{b}+1}$$

$$\Rightarrow\ x = 1 + \frac{1}{y+1} \qquad \left[\because y = \frac{a}{b} \text{and} y^2 > 2 \text{ (given)}\right]$$

Which shows $x^2 < 2\left[\because \frac{1}{y+1} < \text{as } y > 1\right]$

17 **(c)**

Using A.M. ≥ G.M., we have

$$5^{\sin x-1} + 5^{-\sin x-1} \ge 2\sqrt{5^{\sin x-1} \times 5^{-\sin n-1}}$$

$$\Rightarrow 5^{\sin x-1} + 5^{-\sin x-1} \ge \frac{2}{5}$$

18 **(d)**

As we know, if $ax + bx + c > 0$, then $a > 0$ and $D < 0$

$\therefore\ (2)^2 - 4(n-10) < 0 \ \Rightarrow\ n > 11$

9 **(a)**

Since, AM ≥ GM

$$\Rightarrow \frac{bcx + cay + abz}{3} \ge (a^2b^2c^2 \cdot xyz)^{1/3}$$

$$\Rightarrow bcx + cay + abz \ge 3\,abc \quad (\because xyz = abc)$$

20 **(b)**

Since, $\frac{(1+a_1)}{2} \ge \sqrt{1 \cdot a_1} = \sqrt{a_1}$

$$\frac{(1+a_2)}{2} \ge \sqrt{1 \cdot a_2} = \sqrt{a_2}$$

$$\vdots \qquad \vdots$$

$$\frac{(1+a_n)}{2} \ge \sqrt{1 \cdot a_n} = \sqrt{a_n}$$

$$\Rightarrow \frac{1}{2^n}(1+a_1)(1+a_2)\ldots(1+a_n) \ge \sqrt{a_1a_2\ldots a_n} = 1$$

$$\Rightarrow (1+a_1)(1+a_2)\ldots(1+a_n) \ge 2^n$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	B	A	B	A	B	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	A	B	A	B	C	D	A	B