

Topic :-LINEAR INEQUALITIES

1 **(d)**

The LHS of the given inequality is meaningful for $x > 0$ and $x \neq 1$

Now,

$$\log_3 x - \log_x 27 < 2$$

$$\Rightarrow \log_3 x - 3 \log_x 3 < 2$$

$$\Rightarrow \log_3 x - \frac{3}{\log_3 x} < 2$$

$$\Rightarrow \frac{(\log_3 x)^2 - 3 - 2(\log_3 x)}{\log_3 x} < 0$$

$$\Rightarrow \frac{(\log_3 x - 3)(\log_3 x + 1)}{(\log_3 x - 0)} < 0$$

$$\Rightarrow \log_3 x < -1 \text{ or, } 0 < \log_3 x < 3$$

$$\Rightarrow x < 3^{-1} \text{ or, } 3^0 < x < 3^3 \Rightarrow x < \frac{1}{3} \text{ or, } 1 < x < 27$$

Also, $x > 0$ and $x \neq 1$

$$\therefore x \in (0, 1/3) \cup (1, 27)$$

2 **(c)**

Given inequation is $x^2 - 2x + 5 \leq 0$

\therefore Roots are

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm 4i}{2}$$

\therefore Roots are imaginary, therefore no real solutions exist

3 **(b)**

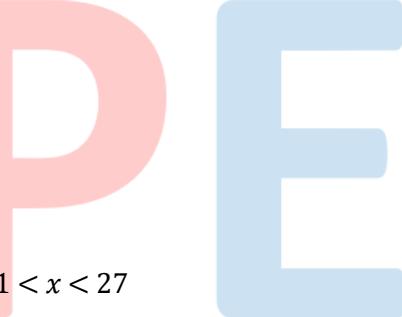
We have,

$$\begin{aligned} \frac{2^{\sin^2 x} + 2^{\cos^2 x}}{2} &\geq \sqrt{2^{\sin^2 x} \times 2^{\cos^2 x}} \quad [\text{Using A.M.} \geq \text{G.M.}] \\ \Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} &\geq 2\sqrt{2} \Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2 \end{aligned}$$

4 **(a)**

Given that, $a^2 + b^2 + c^2 = 1 \quad \dots(i)$

$$\text{Now, } (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$$



$$\Rightarrow 2(ab + bc + ca) \geq -1 \quad [\text{from Eq.(i)}]$$

$$\Rightarrow ab + bc + ca \geq -\frac{1}{2} \dots (\text{iii})$$

$$\text{Also, } a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{1}{2} \{(a-b)^2 + (b-c)^2 + (c-a)^2\} \geq 0$$

$$\Rightarrow ab + bc + ca \leq a^2 + b^2 + c^2$$

$$\Rightarrow ab + bc + ca \leq 1 \quad [\text{from Eq.(i)}] \dots (\text{iii})$$

From relation (ii) and (iii), we get

$$-\frac{1}{2} \leq ab + bc + ca \leq 1$$

5 (b)

We have,

$$\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$$

$$\Rightarrow \sqrt{4x+9} - \sqrt{7x+4} = \sqrt{11x+1}$$

$$\Rightarrow 4x+9 + 7x+4 - 2\sqrt{(4x+9)(7x+4)} = 11x+1$$

$$\Rightarrow -2\sqrt{(4x+9)(7x+4)} = -12$$

$$\Rightarrow (4x+9)(7x+4) = 36$$

$$\Rightarrow 28x^2 + 79x = 0 \Rightarrow x = 0, -\frac{79}{28}$$

Clearly, only $x = 0$ satisfies the given equation

6 (a)

$$\text{Since, } -3 < x + \frac{2}{x} < 3$$

$$\Rightarrow -3 < \frac{(x^2 + 2)x}{x^2} < 3 \quad \Rightarrow -3x^2 < (x^2 + 2)x < 3x^2 (x \neq 0)$$

$$\Rightarrow x(x^2 + 3x + 2) > 0$$

$$\text{And } x(x^2 - 3x + 2) < 0 \quad (x \neq 0)$$

$$\Rightarrow x(x+1)(x+2) > 0$$

$$\text{And } x(x-1)(x-2) < 0$$

$$\Rightarrow x \in (-2, -1) \cup (0, \infty) \dots (\text{i})$$

$$\text{And } x \in (-\infty, 0) \cup (1, 2) \dots (\text{ii})$$

From relations (i) and (ii), we get

$$x \in (-2, -1) \cup (1, 2)$$

7 **(b)**

We have a, b, c are sides of a triangle

$$\therefore b + c - a > 0, c + a - b > 0, a + b - c > 0$$

Let $x = b + c - a, y = c + a - b, z = a + b - c$

$$\Rightarrow y + z = 2a, z + x = 2b, x + y = 2c$$

$$\text{Now, } \frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$$

$$= \frac{y+z}{2x} + \frac{z+x}{2y} + \frac{x+y}{2z}$$

$$= \frac{1}{2} \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{x}{z} + \frac{y}{x} + \frac{z}{y} \right)$$

$$\geq \frac{6}{2} \left(\frac{y}{x} \cdot \frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{y} \cdot \frac{z}{x} \cdot \frac{x}{z} \right) \quad (\because \text{AM} \geq \text{GM})$$

$$= 3$$

8 **(b)**

Let d_1, d_2 be the lengths of diagonals and θ be the angle between them. Then,

$$\text{Area} = \frac{1}{2} d_1 d_2 \sin \theta \Rightarrow a^2 = \frac{1}{2} d_1 d_2 \sin \theta \Rightarrow d_1 d_2 = \frac{2a^2}{\sin \theta}$$

Using A.M. \geq G.M., we have

$$\frac{d_1 + d_2}{2} \geq \sqrt{d_1 d_2} \Rightarrow d_1 + d_2 \geq 2 \sqrt{\frac{2a^2}{\sin \theta}} \geq 2\sqrt{2}a$$

9 **(c)**

We have, $|2x - 3| < |x + 5|$

$$\Rightarrow |2x - 3| - |x + 5| < 0$$

$$\Rightarrow \begin{cases} 3 - 2x + x + 5 < 0, x \leq -5 \\ 3 - 2x - x - 5 < 0, -5 < x \leq \frac{3}{2} \\ 2x - 3 - x - 5 < 0, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x > 8, x \leq -5 \\ x > -\frac{2}{3}, -5 < x \leq \frac{3}{2} \\ x < 8, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2} \right] \cup \left(\frac{3}{2}, 8 \right)$$

$$\Rightarrow x \in \left(-\frac{2}{3}, 8 \right)$$

10 (c)

We have,

$$\begin{aligned} |a_n| &< 2 \text{ i.e. } -2 < a_n < 2 \\ \therefore \max(1 + a_1x + a_2x^2 + \dots + a_nx^n) &= 1 + 2|x| + 2|x|^2 + \dots + 2|x|^n \\ &= 1 + 2|x| \left\{ \frac{1 - |x|^n}{1 - |x|} \right\} \\ &= 1 + 2 \cdot \frac{1}{3} \left\{ \frac{1 - 1/3^n}{1 - 1/3} \right\} > 0 \end{aligned}$$

and,

$$\begin{aligned} \min(1 + a_1x + a_2x^2 + \dots + a_nx^n) &= 1 - 2|x| - 2|x|^2 - \dots - 2|x|^n \\ &= -2[1 + |x| + |x|^2 + \dots + |x|^n] + 3 \\ &= -2 \left\{ \frac{1 - |x|^n}{1 - |x|} \right\} + 3 \\ &= -2 \left\{ \frac{1 - 1/3^n}{1 - 1/3} \right\} + 3 > 0 \end{aligned}$$

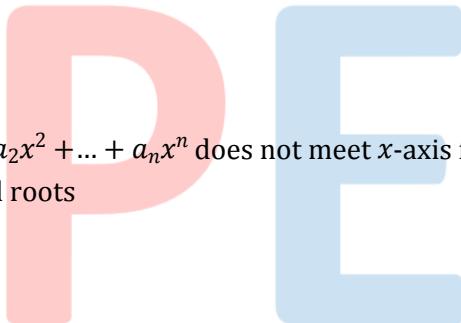
Thus, the curve $y = 1 + a_1x + a_2x^2 + \dots + a_nx^n$ does not meet x -axis for $|x| < 1/3$ and $|a_n| < 2$

Hence, the equation has no real roots

12 (c)

Since, $|r| < 1 \Rightarrow -1 < r < 1$

Also, $a = 5(1 - r)$



$$\Rightarrow 0 < a < 10 \left[\begin{array}{l} \because \text{at } r = -1, a = 0 \\ \text{and at } r = 1, a = 0 \end{array} \right]$$

13 (a)

Consider the curves $y = e^{x-8}$ and $y = 17 - 2x$. These two curves intersect at $(8, 1)$ only. Hence, the equation $e^{x-8} + 2x - 17 = 0$ has exactly one root which is equal to 8

14 (b)

Let $x^2 + 18x + 30 = y$. Then,

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$$

$$\Rightarrow y = 2\sqrt{y + 15}$$

$$\Rightarrow y^2 - 4y - 60 = 0 \Rightarrow (y - 10)(y + 6) = 0 \Rightarrow y = 10$$

$$\therefore x^2 + 18x + 30 = y \Rightarrow x^2 + 18x + 20 = 0$$

\therefore Product of roots = 20

15 (a)

Since, $2 \leq |x - 3| < 4$

$$\Rightarrow 2 \leq x - 3 < 4$$

$$\text{Or } 2 \leq -(x - 3) < 4$$

$$\Rightarrow 5 \leq x < 7 \text{ or } -1 \leq -x < 1$$

$$\Rightarrow 5 \leq x < 7 \text{ or } -1 < x \leq 1$$

$$\therefore x \in (-1, 1] \cup [5, 7)$$

16 (b)

Given that, $x = \left[\frac{a+2b}{a+b} \right]$ and $y = \frac{a}{b}$

$$\therefore x = \frac{a+2b}{a+b} = \frac{\frac{a}{b} + 2}{1 + \frac{a}{b}} = 1 + \frac{1}{\frac{b}{a} + 1}$$

$$\Rightarrow x = 1 + \frac{1}{y+1} \quad [\because y = \frac{a}{b} \text{ and } y^2 > 2 \text{ (given)}]$$

Which shows $x^2 < 2 \left[\because \frac{1}{y+1} < 2 \text{ as } y > 1 \right]$

17 (c)

Using A.M. \geq G.M., we have

$$5^{\sin x-1} + 5^{-\sin x-1} \geq 2\sqrt{5^{\sin x-1} \times 5^{-\sin x-1}}$$

$$\Rightarrow 5^{\sin x-1} + 5^{-\sin x-1} \geq \frac{2}{5}$$

18 (d)

As we know, if $ax + bx + c > 0$, then $a > 0$ and $D < 0$

$$\therefore (2)^2 - 4(n-10) < 0 \Rightarrow n > 11$$

9 (a)

Since, AM \geq GM

$$\Rightarrow \frac{bcx + cay + abz}{3} \geq (a^2 b^2 c^2 \cdot xyz)^{1/3}$$

$$\Rightarrow bcx + cay + abz \geq 3abc \quad (\because xyz = abc)$$

20 (b)

$$\text{Since, } \frac{(1+a_1)}{2} \geq \sqrt{1 \cdot a_1} = \sqrt{a_1}$$

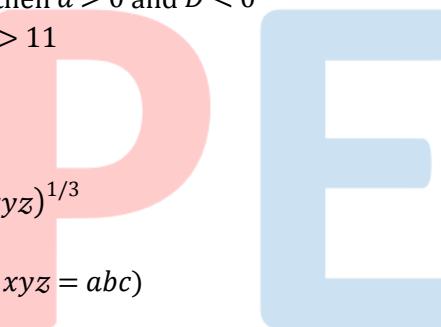
$$\frac{(1+a_2)}{2} \geq \sqrt{1 \cdot a_2} = \sqrt{a_2}$$

$$\vdots \quad \vdots$$

$$\frac{(1+a_n)}{2} \geq \sqrt{1 \cdot a_n} = \sqrt{a_n}$$

$$\Rightarrow \frac{1}{2^n} (1+a_1)(1+a_2) \dots (1+a_n) \geq \sqrt{a_1 a_2 \dots a_n} = 1$$

$$\Rightarrow (1+a_1)(1+a_2) \dots (1+a_n) \geq 2^n$$



ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	B	A	B	A	B	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	A	B	A	B	C	D	A	B

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