DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :5

Topic :-LINEAR INEQUALITIES

1 **(b)**

Using AM $\geq$ GM

$$\frac{\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}}{3} \geq \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \text{...(i)}$$

Again, using AM ≥ GM

$$\frac{a+b}{2} \geq \sqrt{ab}, \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}$$

$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$

$$\Rightarrow \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \leq \frac{1}{2}$$

$\therefore$ From Eq. (i)

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2}$$

82 **(a)**

Two curves $y = e^{|x|}$ and $y = |x|$ does not intersect. So, the equation $e^{|x|} - |x| = 0$ has no solution

3 **(b)**

$$|p-q| = \begin{cases} p-q, p \geq q \\ q-p, p < q \end{cases}$$

$$\therefore \min(p,q) = \begin{cases} p, p < q \\ q, q < p \end{cases}$$

$$\Rightarrow \text{RHS} = \frac{1}{2}(p+q-|p-q|), \text{if } p > q$$

$$\Rightarrow \frac{1}{2}(p+q-p+q) = q$$

and LHS $\min(p,q) = q$

$$\therefore \min(p,q) = \frac{1}{2}(p+q-|p-q|)$$

84 **(a)**

The given equation is

$1+|e^x-1|=e^x(e^x-2)$

$\Rightarrow|e^x-1|+2=(e^x-1)^2$

$\Rightarrow|e^x-1|^2-(e^x-1)-2=0$

$\Rightarrow(|e^x-1|-2)(|e^x-1|+1)=0$

$\Rightarrow|e^x-1|-2=0 \quad [\because |e^x-1|+1\neq 0]$

$\Rightarrow e^x-1=\pm 2 \Rightarrow e^x=3,-1$

$\Rightarrow e^x=3 \Rightarrow x=\log_e 3\,[\because e^x>0 \text{ for all } x]$

5 **(c)**

We have,

$\log_{\sin\frac{2\pi}{3}}(x^2-3x+2)\geq 2$

$\Rightarrow(x^2-3x+2)\leq\left(\frac{\sqrt{3}}{2}\right)^2$ and $x^2-3x+2>0$

$\Rightarrow 4x^2-12x+5\leq 0$ and $x^2-3x+2>0$

$\Rightarrow(2x-1)(2x-5)\leq 0$ and $(x-1)(x-2)>0$

$\Rightarrow\frac{1}{2}\leq x\leq\frac{5}{2}$ and $x<1$ or $x>2$

$\Rightarrow x\in[1/2,1)\cup(2,5/2]$

6 **(c)**

We have, $x^2+6x-27>0$

$\Rightarrow(x+9)(x-3)>0\Rightarrow x<-9$ or $x>3$

$\Rightarrow x\in(-\infty,-9)\cup(3,\infty)$....(i)

And $x^2-3x-4<0$

$\Rightarrow(x-4)(x+1)<0$

$\Rightarrow-1<x<4$....(i)

From relations (i) and (ii), we get

$3<x<4$

7 **(d)**

We have,

$\frac{1}{2}\{(x+y)+|x-y|\}=x$

$\Rightarrow\frac{1}{2}\{(x+y)+|x-y|\}=\frac{1}{2}\{(x+y)+(x-y)\}$

$\Rightarrow|x-y|=x-y\Rightarrow x\geq y$

8 **(c)**

Given, $\frac{4x-1}{3x+1}-1\geq 0$

$\Rightarrow \quad \frac{x-2}{3x+1}\geq 0$

$\Rightarrow\ x-2\geq 0$and$3x+1>0$
Or $x-2\leq 0$ and $3x+1<0$
$\Rightarrow\ x\geq 2$and$x<-\frac{1}{3}$
Or $x\leq 2$ and $x>-\frac{1}{3}$
$\Rightarrow\ x\in\left(-\infty,\ -\frac{1}{3}\right)\cup[2,\infty)$

9 **(c)**

Given inequality holds only, if

$\sin^2\alpha_i=1$ or$\alpha_i=\pm\frac{\pi}{2},\frac{3\pi}{2};\quad(i=2,3,\ldots,n)$

$\Rightarrow$ Number of solutions $=3\times 3\times 3\times\ldots\times(n-1)$times

$=3^{n-1}$

10 **(b)**

We have, $e^x=x(x+1),x<0$

Consider the curves $y=e^x$ and $y=x(x+1)$ for $x<0$. Graphs of these two curve intersect at exactly one point. So, the equation $e^x=x(x+1)$ has exactly one real root

11 **(b)**

Draw graphs of $y=1-x+[x]$ and $y=\frac{1}{x}-\frac{1}{[x]}$

These two curves intersect it infinitely many points

12 **(d)**

We have, $2^x+2^{|x|}\geq 2\sqrt{2}$

Following cases arise:

CASE I$When x\geq 0$

In this case, we have

$2^x+2^x\geq 2\sqrt{2}\Rightarrow 2^x\geq 2^{1/2}\Rightarrow x\geq\frac{1}{2}\Rightarrow x\in\left[\frac{1}{2},\infty\right)$

CASE II$When x<0$

In this case, we have

$2^x+2^{-x}\geq 2\sqrt{2}$
$\Rightarrow(2^x)^2-2\sqrt{2}\times 2^x+1\geq 0$
$\Rightarrow(2^x-\sqrt{2})^2-1\geq 0$
$\Rightarrow(2^x-\sqrt{2}-1)(2^x-\sqrt{2}+1)\geq 0$
$\Rightarrow 2^x\leq\sqrt{2}-1$or, $2^x\geq\sqrt{2}+1$
$\Rightarrow x\leq\log_2(\sqrt{2}-1)$or, $x\geq\log_2(\sqrt{2}+1)$
$\Rightarrow x\leq\log_2(\sqrt{2}-1)\Rightarrow x\in\left(-\infty,\log_2(\sqrt{2}-1)\right)$
Hence, $x\in\left(-\infty,\log_2(\sqrt{2}-1)\right)\cup[1/2,\infty)$

13 **(c)**

$\sin^4\frac{x}{3}+\cos^4\frac{x}{3}>\frac{1}{2}\Rightarrow 1-\frac{1}{2}\sin^2\frac{2x}{3}>\frac{1}{2}$

$\Rightarrow \sin^2\frac{2x}{3}<1\Rightarrow\frac{2x}{3}\in\left(R-(2n+1)\frac{\pi}{2}\right)$

$\Rightarrow x\in R-\left(\frac{3n\pi}{2}+\frac{3\pi}{4}\right);n\in I$

14 **(a)**

We have,

$(\log_5 x)^2+(\log_5 x)<2$

$\Rightarrow(\log_5 x)^2+(\log_5 x)-2<0$

$\Rightarrow(\log_5 x+2)(\log_5 x-1)<0$

$\Rightarrow -2<\ \log_5 x<1\Rightarrow 5^{-1}<x<5\Rightarrow x\in\left(\frac{1}{25},5\right)$

16 **(d)**

$\because\ \sin x+2\sqrt{2}\cos x\geq(\sqrt{3})^2$

$\Rightarrow \sin x+2\sqrt{2}\cos x\geq 3$

$\Rightarrow \sin\left(a+\cos^{-1}\frac{1}{3}\right)\geq 1$

$\Rightarrow\sin\left(x+\cos^{-1}\frac{1}{3}\right)=1(\because \sin x$cannot be greater than 1)

$\therefore x=n\pi+(-1)^n\frac{\pi}{2}-\cos^{-1}\frac{1}{3}$

For solution in the interval $[-2\pi,2\pi]$, $n=0,1,\ -1,\ -2$

17 **(c)**

$x^{(\log_{10}x)^2-3\log_{10}x+1}>1000=10^3$

$\Rightarrow[(\log_{10}x)^2-3\log_{10}x+1]\log_{10}x>3\log_{10}10=3$

$\Rightarrow(\log_{10}x)^3-3(\log_{10}x)^2+\log_{10}x>3$

$\Rightarrow(\log_{10}x)(\log_{10}x-3)+1(\log_{10}x-3)>0$

$\Rightarrow(\log_{10}x-3)(\log_{10}x+1)>0$

$\Rightarrow \log_{10}x-3>0\Rightarrow\log_{10}x>3$

$\Rightarrow x>10^3=1000$

$\Rightarrow x\in(1000,\infty)$

18 **(d)**

$x^{12}-x^{9}+x^{4}-x+1>0$, three cases arise

Case I When $x\leq 0$

$x^{12}>0,\ -x^{9}>0, x^{4}>0,\ -x>0$

$\Rightarrow\ x^{12}-x^{9}+x^{4}+x+1>0, \forall\ x\leq 0$...(i)

Case II When $0<x\leq 1$

$x^{9}<x^{4}, x<1\Rightarrow\ -x^{9}+x^{4}>0and1-x>0$

$\therefore\ x^{12}-x^{9}+x^{4}-x+1>0, \forall\ 0<x\leq 1$...(ii)

Case III When $x>1$

$x^{12}>x^{9}, x^{4}>x$

$\Rightarrow\ x^{12}-x^{9}+x^{4}-x+1>0, \forall\ x>1$...(iii)

$\therefore$ From Eqs. (i), (ii) and (iii) the above equation hold for $x\in R$

20 **(a)**

We have,

$\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

$\Rightarrow x+1+x-1-2\sqrt{x^{2}-1}=4x-1$

$\Rightarrow -2\sqrt{x^{2}-1}=2x-1$

$\Rightarrow 4(x^{2}-1)=4x^{2}-4x+1\Rightarrow 4x-5=0\Rightarrow x=\frac{5}{4}$

This value of x does not satisfy the given equation. So, the equation has no solution

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	A	C	C	D	C	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	C	A	D	D	C	D	D	A