

Topic :-LINEAR INEQUALITIES

1 **(b)**

$$2x - 1 = |x + 7| = \begin{cases} x + 7, & \text{if } x \geq -7 \\ -(x + 7), & \text{if } x < -7 \end{cases}$$

∴ If $x \geq -7$, $2x - 1 = x + 7 \Rightarrow x = 8$

If $x < -7$, $2x - 1 = -(x + 7)$

$$\Rightarrow 3x = -6$$

$\Rightarrow x = -2$, which is not possible

2 **(c)**

Let $f(x) = x^4 - 4x - 1$. Then, the number of changes of signs in $f(x)$ is 1. Therefore, $f(x)$ can have at most one positive real root

We have,

$$f(2) > 0 \text{ and } f(0) < 1$$

Therefore, $f(x)$ has one positive real root between 1 and 2

3 **(c)**

$$\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$$

$$\Rightarrow x^2 - 3x + 2 \leq \frac{3}{4} \quad (\text{If } \log_a b = c \Rightarrow b = a^c)$$

$$\Rightarrow x^2 - 3x + \frac{5}{4} \leq 0$$

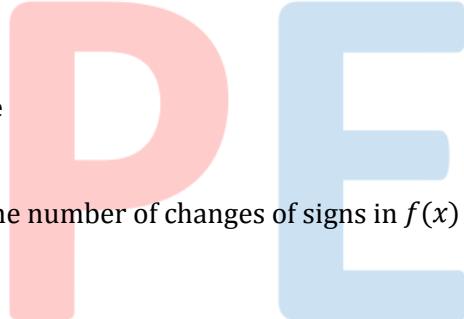
$$\Rightarrow 4x^2 - 12x + 5 \leq 0$$

$$\Rightarrow (2x - 5)(2x - 1) \leq 0$$

$$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2} \dots (i)$$

Also, $x^2 - 3x + 2 > 0$

$$\Rightarrow (x - 1)(x - 2) > 0$$



$$\Rightarrow x < 1 \text{ or } x > 2 \quad \dots(\text{ii})$$

From relation (i) and (ii), we get

$$x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$$

4 (c)

$$\text{Since, } (a-b)^2 + (b-c)^2 + (c-a)^2 \geq 0$$

$$\Rightarrow 2(a^2 + b^2 + c^2) \geq 2(ab + bc + ca)$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} \geq 1$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} + 2 \geq 3$$

Hence, option (c) is correct

5 (b)

$$\text{Given, } (x-1)(x^2 - 5x + 7) < (x-1)$$

$$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$$

$$\Rightarrow (x-1)(x-2)(x-3) < 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (2, 3)$$

6 (d)

Given inequalities are

$$x^2 - 3x - 10 < 0 \text{ and } 10x - x^2 - 16 > 0$$

$$\Rightarrow (x+2)(x-5) < 0 \text{ and } (x-2)(x-8) < 0$$

$$\Rightarrow x \in (-2, 5) \text{ and } x \in (2, 8)$$

$$\Rightarrow x \in (2, 5)$$

7 (a)

$$\log_{1/3}(x^2 + x + 1) < -1 = \log_{1/3}\left(\frac{1}{3}\right)^{-1}$$

$$\Rightarrow x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$$

(∵ where $0 < a < 1$, then $\log_a x < \log_a y \Rightarrow x > y$)

$$\Rightarrow x^2 + x - 2 > 0 \Rightarrow (x+2)(x-1) > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup (1, \infty)$$

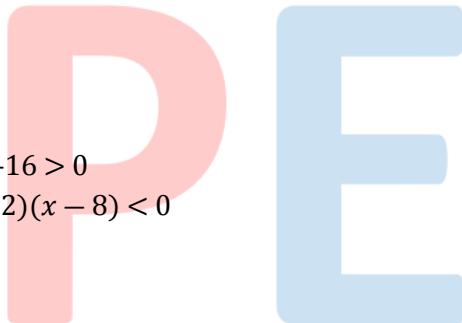
8 (a)

Let (α, β) be a solution of the system for some a . Then, $(-\alpha, \beta)$ is also a solution. So, the system will have unique solution only if

$$\alpha = -\alpha \Rightarrow \alpha = 0$$

Putting $x = \alpha = 0$ and $y = \beta$ in $x^2 + y^2 = 1$, we get $\beta = \pm 1$

Putting $x = \alpha = 0$ and $y = \beta$ in $2^{|x|} + |x| = y + x^2 + a$, we get



$$\beta + a = 1 \Rightarrow a = 1 - \beta$$

$\therefore a = 0$ when $\beta = 1$ and $a = 2$ when $\beta = -1$

CASE I When $a = 0$

In this case, given equations become

$$2^{|x|} + |x| = y + x^2 \text{ and } x^2 + y^2 = 1$$

Now, $x^2 + y^2 = 1 \Rightarrow |x| \leq 1$ and $|y| \leq 2$

$$\therefore 2^{|x|} + |x| = y + x^2 \text{ and } 1 + x^2 \geq y + x^2$$

$$\Rightarrow 2^{|x|} + |x| \leq 1 + x^2$$

$$\Rightarrow 2^{|x|} + |x| \leq 1 + |x| \quad [\because x^2 \leq |x| \text{ when } |x| < 1]$$

$$\Rightarrow x = 0$$

Putting $x = 0$ in $2^{|x|} + |x| = y + x^2$, we get $y = 1$

Thus, for $a = 0$, the system has unique solution $(0, 1)$

CASE II When $a = 2$

In this case, the system of equation is

$$2^{|x|} + |x| = y + x^2 + 2 \text{ and } x^2 + y^2 = 1$$

Clearly, $(0, -1), (1, 0)$ and $(-1, 0)$ satisfy these equations.

So, the system does not have unique solution

9 (c)

We have,

$$\log_{1/3}(x^2 + x + 1) + 1 > 0$$

$$\Rightarrow \log_{1/3}(x^2 + x + 1) > -1$$

$$\Rightarrow x^2 + x + 1 < \left(\frac{1}{3}\right)^{-1}$$

$$\Rightarrow x^2 + x + 1 < 3$$

$$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x+2)(x-1) < 0 \Rightarrow x \in (-2, 1)$$

10 (d)

We have,

$$x^{(\log_{10}x)^2 - 3(\log_{10}x) + 1} > 10^3$$

$$\Rightarrow (\log_{10}x)^2 - 3(\log_{10}x) + 1 > \log_x 10^3$$

$$\Rightarrow (\log_{10}x)^2 - 3(\log_{10}x) + 1 > \frac{3}{\log_{10}x}$$

$$\Rightarrow \frac{(\log_{10}x)^3 - 3(\log_{10}x)^2 + (\log_{10}x) - 3}{\log_{10}x} > 0$$

$$\Rightarrow \frac{(\log_{10}x)^2 + 1}{\log_{10}x} (\log_{10}x - 3) > 0$$

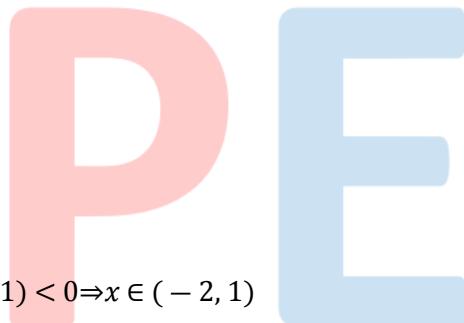
$$\Rightarrow \frac{(\log_{10}x - 3)}{(\log_{10}x - 0)} > 0$$

$$\Rightarrow \log_{10}x < 0 \text{ or, } \log_{10}x > 3$$

$$\Rightarrow x < 1 \text{ or, } x > 10^3$$

$$\Rightarrow x \in (0, 1) \cup (10^3, \infty) \quad [\because \log_{10}x \text{ is defined for } x > 0]$$

11 (a)



Given, $\frac{3 - |x|}{4 - |x|} \geq 0$

$\Rightarrow 3 - |x| \leq 0$ and $4 - |x| < 0$

Or $3 - |x| \geq 0$ and $4 - |x| > 0$

$\Rightarrow |x| \geq 3$ and $|x| > 4$

Or $|x| \leq 3$ and $|x| < 4$

$\Rightarrow |x| > 4$ or $|x| \leq 3$

$\Rightarrow (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$

12 (a)

Now, $3(a^2 + b^2 + c^2) - (a + b + c)^2$

$$= 2(a^2 + b^2 + c^2 - bc - ca - ab)$$

$$= (b - c)^2 + (c - a)^2 + (a - b)^2 \geq 0$$

$\Rightarrow 3(a^2 + b^2 + c^2) \geq (a + b + c)^2 > 9$

$\Rightarrow a^2 + b^2 + c^2 > 3 \Rightarrow (a)$ holds

Now, $a^6 + b^6 \geq 12a^2b^2 - 64$

If $a^6 + b^6 + 64 \geq 12a^2b^2$

i.e., $a^6 + b^6 + 2^6 \geq 3 \cdot 2^2 \cdot a^2b^2$

i.e, if $\frac{a^6 + b^6 + 2^6}{3} \geq (2^6 a^6 b^6)^{1/3}$ ($\because \text{AM} \geq \text{GM}$)

$\Rightarrow (b)$ does not hold



Again, since $\text{AM} \geq \text{HM}$

$$\therefore \frac{a + b + c}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow \frac{\alpha}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{9}{\alpha}$$

$\Rightarrow (c)$ does not hold

13 (c)

Using A.M. \geq G.M., we have

$$\frac{a_1 + a_2 + a_3}{3} \geq (a_1 a_2 a_3)^{1/3} \text{ and } \frac{1}{3} \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq \left(\frac{1}{a_1 a_2 a_3} \right)^{1/3}$$

$$\Rightarrow (a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq 9$$

14 (a)

We have,

$$x - \frac{1}{x} = B \text{ and } x^2 + \frac{1}{x^2} = A$$

$$\therefore \left(x - \frac{1}{x} \right)^2 = B^2$$

$$\Rightarrow A - 2 = B^2 \Rightarrow A = B^2 + 2 \Rightarrow \frac{A}{B} = B + \frac{2}{B}$$

But, A.M. \geq G.M.

$$\Rightarrow B + \frac{2}{B} \geq 2 \sqrt{B \times \frac{2}{B}} \Rightarrow B + \frac{2}{B} \geq 2\sqrt{2} \Rightarrow \frac{A}{B} \geq 2\sqrt{2}$$

Hence, the minimum value of $\frac{A}{B}$ is $2\sqrt{2}$

15 (c)

As discussed in the above problem, if n is odd, there is one change of sign in (i). Therefore, $f(x)$ can have at most one negative real root. In this case, we have

$$f(-1) = -2n - 2 < 0, f(0) = 1 > 0$$

So, the negative real root lies between -1 and 0

16 (c)

$$\text{Given, } {}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 50$$

$$\Rightarrow \frac{(n-1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-1)!} \leq 50$$

$$\Rightarrow \frac{(n+1)!}{3!} \left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!} \right] \leq 50$$

$$\Rightarrow (n+1)! \left(\frac{n-1-3}{(n-1)!} \right) \leq 300$$

$$\Rightarrow (n+1)n(n-4) \leq 300$$

For $n = 8$, it satisfies the above inequality

But $n = 1$ does not satisfy the above inequality

17 (d)

We have,

$$f(\theta) = \sec^2 \theta + \cos^2 \theta = (\sec \theta - \cos \theta)^2 + 2 \geq 2$$

$$\Rightarrow f(\theta) \in [2, \infty)$$

18 (d)

Let a_1, a_2, \dots, a_n be n positive integers such that

$$a_1 a_2 \dots a_n = n^n$$

Since, AM \geq GM

$$\therefore \frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 \dots a_n)^{1/n}$$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq n$$

$$\Rightarrow a_1 + a_2 + \dots + a_n \geq n^2$$

19 **(a)**

$$\log_2(x^2 - 3x + 18) < 4$$

$$\Rightarrow x^2 - 3x + 18 < 16 \quad (\text{If } \log_a b < c \Rightarrow b < a^c)$$

$$\Rightarrow x^2 - 3x + 2 < 0$$

$$\Rightarrow (x-1)(x-2) < 0$$

$$\Rightarrow x \in (1, 2)$$

20 **(d)**

We have,

$$[x]^2 = [x+6]$$

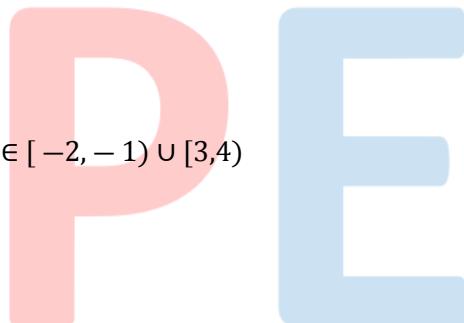
$$\Rightarrow [x]^2 = [x] + 6$$

$$\Rightarrow [x]^2 - [x] - 6 = 0$$

$$\Rightarrow ([x]-3)([x]+2) = 0$$

$$\Rightarrow [x] = 3, [x] = -2$$

$$\Rightarrow x \in [3, 4] \text{ or } x \in [-2, -1] \Rightarrow x \in [-2, -1] \cup [3, 4]$$



ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	C	B	D	A	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	A	C	C	D	D	A	D

P
E