DPP
DAILY PRACTICE PRODUCTS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :4

Topic :-LINEAR INEQUALITIES

1 **(b)**

$2x - 1 = |x + 7| = \begin{cases} x + 7, \text{if } x \geq -7 \\ -(x + 7), \text{if } x < -7 \end{cases}$

$\therefore$ If $x \geq -7$, $2x - 1 = x + 7 \Rightarrow x = 8$

If $x < -7$, $2x - 1 = -(x + 7)$

$\Rightarrow 3x = -6$

$\Rightarrow x = -2$, which is not possible

2 **(c)**

Let $f(x) = x^4 - 4x - 1$. Then, the number of changes of signs in $f(x)$ is 1. Therefore, $f(x)$ can have at most one positive real root

We have,

$f(2) > 0 \text{and} f(0) < 1$

Therefore, $f(x)$ has one positive real root between 1 and 2

3 **(c)**

$\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$

$\Rightarrow x^2 - 3x + 2 \leq \frac{3}{4}$ (If $\log_a b = c \Rightarrow b = c^a$)

$\Rightarrow x^2 - 3x + \frac{5}{4} \leq 0$

$\Rightarrow 4x^2 - 12x + 5 \leq 0$

$\Rightarrow (2x - 5)(2x - 1) \leq 0$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2}$...(i)

Also, $x^2 - 3x + 2 > 0$

$\Rightarrow (x - 1)(x - 2) > 0$

$\Rightarrow x < 1$ or $x > 2$ ...(ii)

From relation (i) and (ii), we get

$x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

4 **(c)**

Since, $(a-b)^2 + (b-c)^2 + (c-a)^2 \geq 0$

$\Rightarrow 2(a^2 + b^2 + c^2) \geq 2(ab + bc + ca)$

$\Rightarrow \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} \geq 1$

$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} + 2 \geq 3$

Hence, option (c) is correct

5 **(b)**

Given, $(x-1)(x^2 - 5x + 7) < (x-1)$

$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$

$\Rightarrow (x-1)(x-2)(x-3) < 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, 3)$

6 **(d)**

Given inequalities are

$x^2 - 3x - 10 < 0$ and $10x - x^2 - 16 > 0$

$\Rightarrow (x+2)(x-5) < 0$ and $(x-2)(x-8) < 0$

$\Rightarrow x \in (-2, 5)$ and $x \in (2, 8)$

$\Rightarrow x \in (2, 5)$

7 **(a)**

$\log_{1/3}(x^2 + x + 1) < -1 = \log_{1/3}\left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$

($\because$ where $0 < a < 1$, then $\log_a x < \log_a y \Rightarrow x > y$)

$\Rightarrow x^2 + x - 2 > 0 \Rightarrow (x+2)(x-1) > 0$

$\Rightarrow x \in (-\infty, -2) \cup (1, \infty)$

8 **(a)**

Let (α, β) be a solution of the system for some a. Then, $(-\alpha, \beta)$ is also a solution. So, the system will have unique solution only if

$\alpha = -\alpha \Rightarrow \alpha = 0$

Putting $x = \alpha = 0$ and $y = \beta$ in $x^2 + y^2 = 1$, we get $\beta = \pm 1$

Putting $x = \alpha = 0$ and $y = \beta$ in $2^{|x|} + |x| = y + x^2 + a$, we get

$\beta + a = 1 \Rightarrow a = 1 - \beta$

$\therefore a = 0$ when $\beta = 1$ and $a = 2$ when $\beta = -1$

<u>CASE I</u> When $a = 0$

In this case, given equations become

$2^{|x|} + |x| = y + x^2$ and $x^2 + y^2 = 1$

Now, $x^2 + y^2 = 1 \Rightarrow |x| \leq 1$ and $|y| \leq 2$

$\therefore 2^{|x|} + |x| = y + x^2$ and $1 + x^2 \geq y + x^2$

$\Rightarrow 2^{|x|} + |x| \leq 1 + x^2$

$\Rightarrow 2^{|x|} + |x| \leq 1 + |x| \quad [\because x^2 \leq |x| \text{ when } |x| < 1]$

$\Rightarrow x = 0$

Putting $x = 0$ in $2^{|x|} + |x| = y + x^2$, we get $y = 1$

Thus, for $a = 0$, the system has unique solution (0, 1)

<u>CASE II</u> When $a = 2$

In this case, the system of equation is

$2^{|x|} + |x| = y + x^2 + 2$ and $x^2 + y^2 = 1$

Clearly, $(0, -1)$, $(1, 0)$ and $(-1, 0)$ satisfy these equations.

So, the system does not have unique solution

9 **(c)**

We have,

$\log_{1/3}(x^2 + x + 1) + 1 > 0$

$\Rightarrow \log_{1/3}(x^2 + x + 1) > -1$

$\Rightarrow x^2 + x + 1 < \left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x + 2)(x - 1) < 0 \Rightarrow x \in (-2, 1)$

10 **(d)**

We have,

$x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 10^3$

$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \log_x 10^3$

$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \frac{3}{\log_{10} x}$

$\Rightarrow \frac{(\log_{10} x)^3 - 3(\log_{10} x)^2 + (\log_{10} x) - 3}{\log_{10} x} > 0$

$\Rightarrow \frac{\{(\log_{10} x)^2 + 1\}(\log_{10} x - 3)}{\log_{10} x} > 0$

$\Rightarrow \frac{(\log_{10} x - 3)}{(\log_{10} x - 0)} > 0$

$\Rightarrow \log_{10} x < 0$ or, $\log_{10} x > 3$

$\Rightarrow x < 1$ or, $x > 10^3$

$\Rightarrow x \in (0,1) \cup (10^3, \infty)$ $[\because \log_{10} x$ is defined for $x > 0]$

11 **(a)**

Given, $\frac{3-|x|}{4-|x|} \geq 0$
$\Rightarrow \; 3-|x| \leq 0$ and $4-|x|<0$
Or $3-|x| \geq 0$ and $4-|x|>0$
$\Rightarrow \; |x| \geq 3$ and $|x|>4$
Or $|x| \leq 3$ and $|x|<4$
$\Rightarrow \; |x|>4$ or $|x| \leq 3$
$\Rightarrow(-\infty,-4) \cup[-3,3] \cup(4, \infty)$
12 **(a)**
Now, $3\left(a^2+b^2+c^2\right)-(a+b+c)^2$

$=2\left(a^2+b^2+c^2-b c-c a-a b\right)$

$=(b-c)^2+(c-a)^2+(a-b)^2 \geq 0$

$\Rightarrow 3\left(a^2+b^2+c^2\right) \geq(a+b+c)^2>9$

$\Rightarrow a^2+b^2+c^2>3 \Rightarrow$ (a)holds

Now, $a^6+b^6 \geq 12 a^2 b^2-64$

If $a^6+b^6+64 \geq 12 a^2 b^2$

$ie, \;\; a^6+b^6+2^6 \geq 3 \cdot 2^2 \cdot a^2 b^2$

$ie,\text{if} \frac{a^6+b^6+2^6}{3} \geq\left(2^6 a^6 b^6\right)^{1/3} \quad (\because \text{AM} \geq \text{GM})$

$\Rightarrow(b)$does not hold

Again, since AM $\geq$ HM

$\therefore \frac{a+b+c}{3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \frac{\alpha}{3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{9}{\alpha}$

$\Rightarrow(c)$ does not hold

13 **(c)**
Using A.M. $\geq$ G.M., we have
$\frac{a_1+a_2+a_3}{3} \geq\left(a_1 a_2 a_3\right)^{1/3}$ and $\frac{1}{3}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right) \geq\left(\frac{1}{a_1 a_2 a_3}\right)^{1/3}$

$\Rightarrow(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)\geq 9$

14 **(a)**

We have,

$x-\frac{1}{x}=B$ and $x^2+\frac{1}{x^2}=A$

$\therefore\left(x-\frac{1}{x}\right)^2=B^2$

$\Rightarrow A-2=B^2\Rightarrow A=B^2+2\Rightarrow\frac{A}{B}=B+\frac{2}{B}$

But, A.M. $\geq$ G.M.

$\Rightarrow B+\frac{2}{B}\geq 2\sqrt{B\times\frac{2}{B}}\Rightarrow B+\frac{2}{B}\geq 2\sqrt{2}\Rightarrow\frac{A}{B}\geq 2\sqrt{2}$

Hence, the minimum value of $\frac{A}{B}$ is $2\sqrt{2}$

15 **(c)**

As discussed in the above problem, if n is odd, there is one change of sign in (i). Therefore, $f(x)$ can have at most one negative real root. In this case, we have

$f(-1)=-2n-2<0, f(0)=1>0$

So, the negative real root lies between -1 and 0

16 **(c)**

Given, ${}^{n+1}C_{n-2}-{}^{n+1}C_{n-1}\leq 50$

$\Rightarrow\frac{(n-1)!}{3!(n-2)!}-\frac{(n+1)!}{2!(n-1)!}\leq 50$

$\Rightarrow\frac{(n+1)!}{3!}\left[\frac{1}{(n-2)!}-\frac{3}{(n-1)!}\right]\leq 50$

$\Rightarrow(n+1)!\left(\frac{n-1-3}{(n-1)!}\right)\leq 300$

$\Rightarrow(n+1)n(n-4)\leq 300$

For $n=8$, it satisfy to the above inequality

But $n=1$ it does not satisfy the above inequality

17 **(d)**

We have,

$f(\theta)=\sec^2\theta+\cos^2\theta=(\sec\theta-\cos\theta)^2+2\geq 2$

$\Rightarrow f(\theta)\in[2,\infty)$

18 **(d)**

Let $a_1, a_2, \ldots, a_n$ be n positive integers such that

$a_1a_2\ldots a_n=n^n$

Since, AM $\geq$ GM

$\therefore\frac{a_1+a_2+\ldots+a_n}{n}\geq(a_1a_2\ldots a_n)^{1/n}$

$\Rightarrow \frac{a_1 + a_2 + \ldots + a_n}{n} \geq n$

$\Rightarrow a_1 + a_2 + \ldots + a_n \geq n^2$

19 **(a)**

$\log_2\left(x^2 - 3x + 18\right) < 4$

$\Rightarrow x^2 - 3x + 18 < 16$ (If $\log_a b < c \Rightarrow b < a^c$)

$\Rightarrow x^2 - 3x + 2 < 0$

$\Rightarrow (x-1)(x-2) < 0$

$\Rightarrow x \in (1, 2)$

20 **(d)**

We have,

$[x]^2 = [x+6]$

$\Rightarrow [x]^2 = [x] + 6$

$\Rightarrow [x]^2 - [x] - 6 = 0$

$\Rightarrow ([x]-3)([x]+2) = 0$

$\Rightarrow [x] = 3, [x] = -2$

$\Rightarrow x \in [3,4)$ or $x \in [-2, -1) \Rightarrow x \in [-2, -1) \cup [3,4)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	C	C	B	D	A	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	A	C	C	D	D	A	D